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A Gray Level-Gradient-Based Two-Dimensional (2D) Otsu Thresholding Method for Image Segmentation

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The traditional gray level two-dimensional (2D) histogram region-dependent method often produces over-segmentation and cannot meet the increasing demands of image segmentation. We propose a gray level gradient 2D histogram segmentation method. Furthermore, we put forward an improved 2D histogram Otsu thresholding method and a fast, recursive algorithm. The results show that the improved Otsu's method creates better segmentation results, extracts the object with a clearer boundary, obtains better homogeneity in the region, and has great resistance to noises. Compared to the traditional 2D Otsu method, the improved Otsu method has more advantages, including faster running speed and better segmentation results.

Keywords: Otsu's method, two-dimensional histogram, thresholding method, gray level-gradient

1. INTRODUCTION

Image segmentation is an important step in the process of image processing, and the thresholding method plays a major role in image segmentation due to its simplicity and better visualization [1]. The largest between-class variance proposed by Nobuyuki Otsu, a Japanese researcher, has been widely used in the thresholding method and is also one of the hot topics in the study of thresholding segmentation algorithms [2]. The Otsu method determines the optimal threshold value based on the maximum between-class variance between the object and background, but it cannot reflect the spatial relationship between pixels and accurately segment the images corrupted by noises. Based on the 1D histogram, Liu et al. propose a 2D histogram-based Otsu thresholding method, which utilizes both the gray level of each pixel and the average gray level of its neighborhood to construct a 2D histogram to produce better image segmentation results [3]. However, this method has several shortcomings, such as high computational complexity and long running time of an algorithm. To reduce the running time, He et al. developed a fast algorithm that divides a 2D histogram into four regions using two cross lines parallel to each axis, and the threshold value is determined based on two regions along the diagonal [4]. However, this method still has some limitations since it assumes that the probability of the gray level of a pixel near the threshold and the average gray level of its neighborhood is negligible (zero), which is unrealistic.

In this paper, we propose a 2D histogram segmentation method based on the gray level gradients in the neighborhood and further put forward an improved 2D histogram-based Otsu thresholding method and a fast, recursive algorithm.

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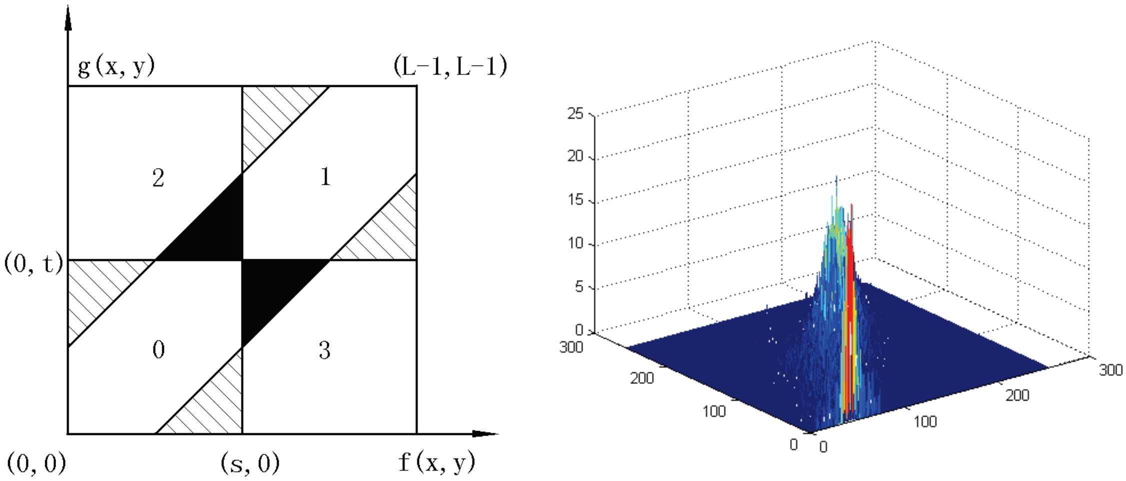


(a) Regional division of a 2D histogram

(b) 2D histogram of the image of bacteria

Figure 1 Traditional gray level: average gray-level-based 2D histogram

2. 2D HISTOGRAM

2.1 Traditional 2D Histogram

The regional division of a 2D histogram is shown in Fig. 1(a). The average gray level of the neighborhood of each pixel was computed to obtain the smoothed image g(x. y) [5]. As shown in Fig. 1(a), let the pixels of an image be represented in L gray levels, the traditional 2D histogram is divided into four regions. The x coordinate $f(x,\ y)$ represents the gray level of a given pixel, the y coordinate $g(x,\ y)$ represents the average gray level of pixels in its neighborhood, and the threshold vector is $(s,\ t)$.

Fig. 1(a) shows that, in regions 0 and 1, the gray level of the interior pixels is very close to the average gray level of their neighborhood, so region 0 and region 1 can be used to represent the object and background, respectively [6]. In contrast, the gray level of the interior pixels in regions 2 and 3 is significantly different from the average gray level of their neighborhood; hence, region 2 and region 3 can be considered as the edge or noise. In the slash-hatched areas in regions 0 and 1 (Fig. 1(a)), the relatively large difference between the gray level of the pixel and the average gray level of its neighborhood leads to the wrong regional division; i.e., the pixels in the edge and noisy areas are incorrectly classified as pixels in the object or background [7]. However, in the dark areas in regions 2 and 3 (Fig. 1 (a)), because the gray level of the pixel is close to the average gray level of its neighborhood, the pixels in the object or background are wrongly considered as edge pixels and noisy pixels [8]. It appears that the wrong regional division of the 2D histogram obtained by the traditional method cannot differentiate the pixels in the object and background from those in the edge and noisy areas, thereby producing the undesirable image segmentation results.

Based on the traditional 2D histogram, we propose an improved method for dividing the region of a 2D histogram by replacing the average gray level of a pixel's neighborhood with the gray level gradient of its neighboring pixels, taking full advantage of information about edges in the image and other specific features [9]. The proposed method can greatly improve the segmentation result.

2.2 Improved 2D Histogram

In the improved 2D histogram, the y coordinate represents the absolute value of the difference between the gray level of a pixel and the average gray level of its neighborhood, $|f(x,\ y) - g(x,\ y)|$, instead of g(x,y) used in the traditional 2D histogram [10]. The x coordinate $f(x,\ y)$ represents the gray level of a pixel. For a given pixel in the image, we obtain a pair $(i,\ j)$ with i representing the gray level of a pixel and j representing the gray level gradient of its neighborhood, respectively. Then, the joint probability of this pair is given by the following equation (Eq. 1):

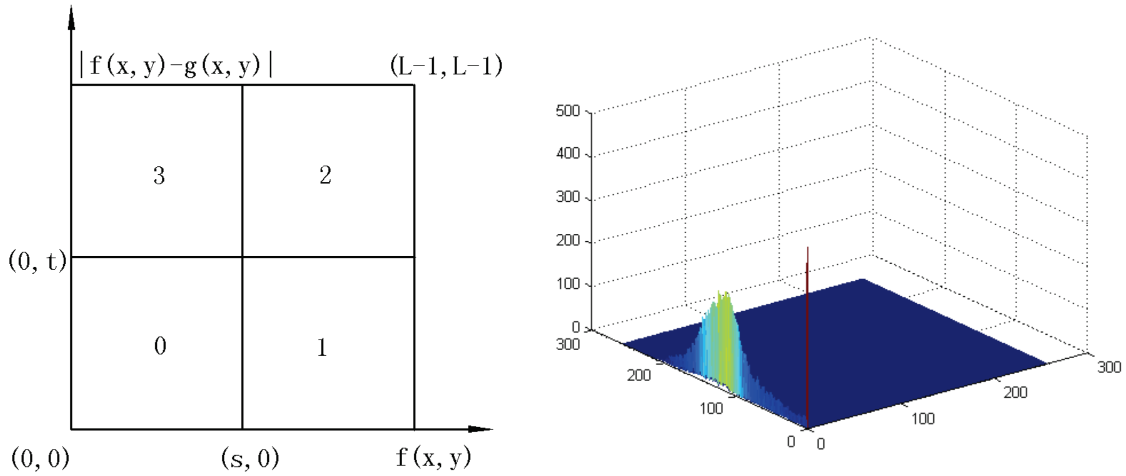
$$p_{ij} = \frac{c_{ij}}{M \times N}, 0 \le i,\ j \le L-1 \tag{1}$$

where $\sum_{i=0}^{L-1} \sum_{j=0}^{L-1} p_{ij} = 1, c_{ij}$ is the number of the occurrence of the pair $(i,\ j)$ and defined as

$$c_{ij} = \sum_{x=1}^{M} \sum_{y=1}^{N} \delta(f(x,\ y) - i)\delta(|f(x, y) - g(x,\ y)| - j),$$
$$i = 0, 1, \ldots,\ L-1,\ j = 0, 1, \ldots, L-1, \tag{2}$$

For a given threshold vector $(s,\ t)$, it can divide the 2D histogram into four regions, as shown in Fig.2(a).

The gray level gradient indicates the difference in gray levels of the pixels in the neighborhood. Fig.2(a) shows that, in the interior of the object (region 0) and background (region 1), because the difference between the gray level of a pixel and the average gray level of its neighboring pixels is relatively small, the gradient of the neighboring pixels is also small [11]. Conversely, because the gray level of a pixel on the edges of the object or background is significantly different from the average gray level of its neighboring pixels, which produces the big gradient of the neighboring pixels, region 2 and region 3 can be used to represent the edge and noise,



(a)Regional division

(b) 2D histogram of the image of bacteria

Figure 2 The proposed gray level-gradient based 2D histogram

respectively [12]. This method of dividing the 2D histogram ensures that regions 0 and 1 contain as many pixels as possible, and simultaneously overcomes the deficiency of the traditional method of dividing a histogram in which regions 0 and 1 contain noisy pixels.

3. OTSU METHOD BASED ON THE IMPROVED 2D HISTOGRAM

Let two classes of C_0 and C_1 represent the object and background, respectively. If P_0 and P_1 represent the probabilities of occurrence of these two classes at a vector $(s,\ t)$, P_0 and P_1 are defined as $P_0(s,\ t) = \sum_{i=0}^{s}\sum_{j=0}^{t} p_{ij}$ and $P_1(s,\ t) = \sum_{i=s+1}^{L-1}\sum_{j=0}^{t} p_{ij}$, respectively. The average vectors of μ_0 and μ_1 corresponding to these two classes are defined as the following equations (Eq.3 and Eq.4), respectively.

$$\mu_0(s,\ t) = (\mu_{0,0},\ \mu_{0,1})^{\mathrm{T}} = \left(\sum_{i=0}^{s}\sum_{j=0}^{t}\frac{ip_{ij}}{P_0},\ \sum_{i=0}^{s}\sum_{j=0}^{t}\frac{jp_{ij}}{P_0}\right)^{\mathrm{T}} \tag{3}$$

$$\mu_1(s, t) = (\mu_{1,0}, \mu_{1,1})^{\mathrm{T}} = \left(\sum_{i=s+1}^{L-1}\sum_{j=0}^{t}\frac{ip_{ij}}{P_1}, \sum_{i=s+1}^{L-1}\sum_{j=0}^{t}\frac{jp_{ij}}{P_1}\right)^{\mathrm{T}} \tag{4}$$

The total mean vector (μ_T) of the 2D histogram is defined as

$$\mu_T = (\mu_{T,0},\ \mu_{T,1})^{\mathrm{T}} = \left(\sum_{i=0}^{L-1}\sum_{j=0}^{L-1} ip_{ij},\ \sum_{i=0}^{L-1}\sum_{j=0}^{L-1} jp_{ij}\right)^{\mathrm{T}} \tag{5}$$

If the sum of the probabilities of regions 2 and 3 is close to zero, the between-class scatter matrix $S_B(s,\ t)$ is defined as

$$\mathbf{S}_B(s,\ t) = \frac{(P_0\mu_{T,0} - P_{0,0})^2 + (P_0\mu_{T,1} - P_{0,1})^2}{P_0(1-P_0)} \tag{6}$$

where

$$P_{0,0}(s,\ t) = \sum_{i=0}^{s}\sum_{j=0}^{t} ip_{ij} \tag{7}$$

$$P_{0,1}(s,\ t) = \sum_{i=0}^{s}\sum_{j=0}^{t} jp_{ij} \tag{8}$$

The trace of $\mathbf{S}_B$ is used to measure the between-class variance and defined as

$$\begin{aligned} Tr\mathbf{S}_B(s,\ t) &= P_0\left[(\mu_{0i} - \mu_{Ti})^2 + (\mu_{0j} - \mu_{Tj})^2\right] \\ &\quad + P_1\left[(\mu_{1i} - \mu_{Ti})^2 + (\mu_{1j} - \mu_{Tj})^2\right] \\ &= \frac{\left[(P_0\mu_{Ti} - \mu_i)^2 + (P_0\mu_{Tj} - \mu_j)^2\right]}{P_0(1-P_0)} \end{aligned} \tag{9}$$

where μ_i and μ_j are the zero derivative of the vector of the object C_0.

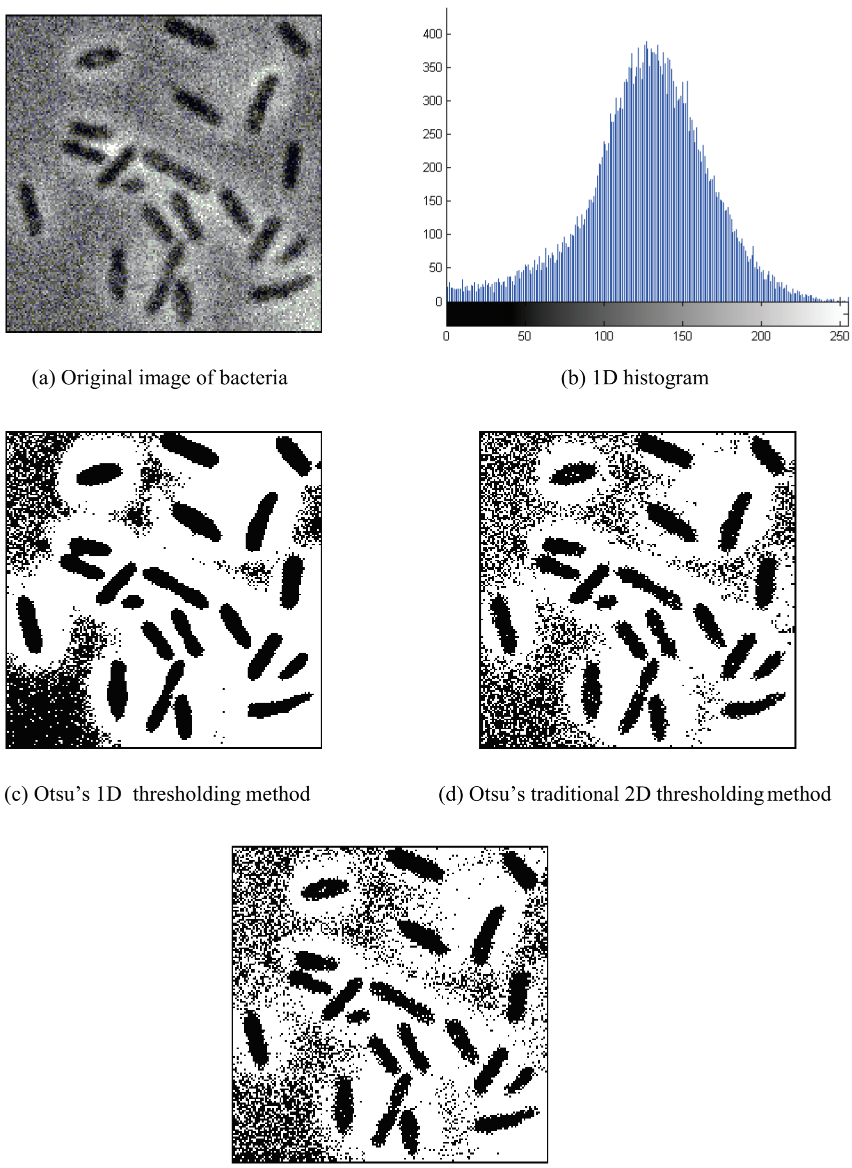
$$\mu_i = \sum_{i=0}^{s}\sum_{j=0}^{t} ip_{ij} \tag{10}$$

$$\mu_j = \sum_{i=0}^{s}\sum_{j=0}^{t} jp_{ij} \tag{11}$$

The optimal threshold $(s^*,\ t^*)$ is determined by maximizing the trace of $S_B(s,\ t)$,

$$(S^*,\ t^*) = \arg\max_{0\le(s,t)\le L-1}\{Tr\mathbf{S}_B(s,\ t)\} \tag{12}$$

The optimal threshold $(s^*,\ t^*)$ can be obtained after searching the entire solution space (L-l) × (L-l). However, a large amount of computation and high computational complexity cannot meet the requirements for real-time applications. Therefore, we propose a fast, recursive algorithm to increase the computational speed and reduce the computational complexity.



(a) Original image of bacteria

(b) 1D histogram

(c) Otsu's 1D thresholding method

(d) Otsu's traditional 2D thresholding method

(e) Improved gray level gradient 2D Otsu thresholding method

Figure 3 Comparison of different segmentation results of the image corrupted with Gaussian noise

The recursive algorithms for μ_i are given below

$$\begin{aligned}
P_0(0,0) &= P_{00}\\
P_0(s,0) &= P_0(s-1,0) + P_{s0}\\
P_0(0,\ t) &= P_0(0,\ t-1) + P_{0t}\\
P_0(s,\ t) &= P_0(s-1,\ t) + P_0(s,\ t-1)\\
&\quad - P_0(s-1,\ t-1) + P_{st} \qquad (13)
\end{aligned}$$

$$\begin{aligned}
\mu_i(0,0) &= 0\\
\mu_i(s,\ 0) &= \mu_i(s-1,0) + s\cdot P_{s0}\\
\mu_i(0,\ t) &= \mu_i(0,\ t-1) + 0\cdot P_{0t}\\
\mu_i(s,\ t) &= \mu_i(s-1,\ t) + \mu_i(s,\ t-1)\\
&\quad - \mu_i(s-1,\ t-1) + s\cdot P_{st} \qquad (14)
\end{aligned}$$

Similarly, the recursive algorithms for μ_j can be derived.

By using these recursive algorithms, the method reduces the number of dimensions of the solution space, obtains the optimal threshold $(s^*,\ t^*)$ without searching throughout the whole solution space, and reduces the computational complexity from $O(L^4)$ to $O(L^2)$.

4. RESULTS AND DISCUSSION

The two methods, including the improved 2D Otsu method based on gray level gradients and the traditional Otsu method, are applied to segment the image of bacteria corrupted by Gaussian noise, using Matlab 7.2 on an Intel Core™ Dual-Core 2.93 GHz microprocessor. The gray level gradients in the neighborhood around each pixel are taken over a 3 × 3 window The results are shown in Fig.3.

It is found that the traditional 2D Otsu method cannot segment those regions with a great variance in the gray level, which could be mistakenly identified as small objects;

Table 1 Comparison of running time and thresholds of two 2D Otsu's method.

	Threshold	Running time(s)
1D Otsu's thresholding method	117	0.56
Traditional 2D Otsu's thresholding method [4]	(116, 133)	2.54
Improved 2D Otsu's thresholding method	(116, 172)	1.61

however, it can better segment big objects. As shown in Fig.3(e), the improved 2D Otsu method based on the gray level gradient can obtain the objects with relatively clear edges and better homogeneity within the region. It demonstrates that the proposed method produces a better segmentation result, has fast computational speed, and can better satisfy the requirements of real-time applications, as shown in Table 1.

5. CONCLUSIONS

In this work, we propose an improved 2D histogram based on gray level gradients and a method for dividing the region. Furthermore, based on the 2D histogram, we improve the Otsu thresholding method and derive fast recursive algorithms. The results show that the proposed algorithms produce better segmentation results, obtain the object with a clearer boundary, and have better resistance to noises. Compared to the traditional 2D Otsu algorithm, the proposed method has a faster computational speed and a better segmentation result.

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