

Simulation Analysis of Random Initial Error with Iterative Learning Control Method for Robot Arms

Zhengjie Lu^{1*}, Mengji Chen^{1†} and Yinjun Zhang^{1,2‡}

¹School of Mechanical and Electrical Engineering, Hechi University, Yizhou, Guangxi, China

²School of mechanical and electrical engineering, Guangxi Science & Technology Normal University, Laibin, China

In this paper, Iterative Learning Control (ILC) is used as the core algorithm. By improving ILC algorithm, a control algorithm suitable for trajectory tracking of industrial robots is proposed. Without resetting the initial conditions, an iterative learning control method is designed to accelerate the suppression of random initial state errors. A modified initial state error interval is defined, which decreases with the number of iterations. Combining with the iterative learning control algorithm, the industrial robot can track the trajectory without resetting the initial conditions, and the tracking error converges to zero asymptotically. In terms of A norm, the convergence of the iterative learning control algorithm is proved. The simulation experiment results of the iterative learning control algorithm for accelerating the suppression of random initial state error are given, and compared with the simulation experiment results of the iterative learning control method without acceleration suppression random initial state error. The results show that the proposed condition-free acceleration is effective. The iterative learning control method for suppressing the random initial state error has a good inhibitory effect on the random initial error of industrial robots.

Keywords: Industrial robots, iterative learning control, disturbance

1. INTRODUCTION

Industrial robots have become an important part of industrial development. They can replace heavy and repetitive human labor, and are an indispensable piece of equipment for large-scale and accurate manufacturing, as well as being deployed in dangerous, harsh and extreme working environments. In China in recent years, there has been a rapid development in the field of industrial robots. In 2013, the sales of industrial robots in China surpassed those of Japan for the first time, and China became the world's largest manufacturer and vendor of industrial robots [1]. In 2018, China's industrial robot sales have reached 156,400 units, ranking first in the world for five consecutive years. Although the sales and retention of industrial robots rank first, there are 478 industrial robots for

every 10,000 workers in Korea, and the density of industrial robots in China is only 36, which is more than ten times less than Korea, so the future industrial robots market in China is huge [2].

The control of the trajectory of industrial robots is an important part of industrial automation, ensuring that the tracking error of industrial robots is reduced to zero, which guarantees that the mechanical arm system can follow the desired trajectory established by humans [3]. As a typical time-varying, highly nonlinear and strongly coupled dynamic system, it is difficult for industrial robot systems to obtain accurate dynamic models in actual modeling [4]. It is especially important to choose appropriate control algorithms.

Japanese scholar Uchiyama first proposed the idea of Iterative Learning Control (ILC) in 1978 [5]. The iterative learning control method is widely used for the control of industrial robots. This method enables the control system to self-learn and self-improve. Iterative learning control is applied to controlled systems with repetitive motion properties

*116185047@qq.com

†782723236@qq.com

‡txeb@163.com

to achieve full tracking of tasks at finite intervals [5,6]. By controlling the control system, the undesired control signal is corrected by the deviation of the output signal from the given target, so that the tracking performance of the system is improved. The study of iterative learning control is very important for dynamic systems with strong nonlinear coupling, high positional repetitiveness, difficult modeling and high-precision trajectory tracking control requirements [7].

In the control system, the initial value conditions directly affect the convergence of the system. Using a restricted equivalent form of the singular system, TianSenping scholars proposed an open-loop PD-type iterative learning law to study the state tracking problem of the system [8]. At present, in many research studies on iterative learning control, it is assumed that the initial state of each industrial robot is exactly the same as the initial state of the desired trajectory. However, in terms of practical application, the two cannot be completely equal. The initial value problem has been studied by many scholars. Heinzinger and other scholars have given the D-type iterative learning control that introduces the forgetting factor in order to eliminate the influence of the initial deviation on the system [9]. Arimoto and other scholars further discussed the influence of the initial deviation size on the convergence of the iterative learning algorithm, and the forgetting factor is essential to the algorithm convergence [5,10]. In 1991, Lee et al. pointed out that the cause of system instability is the difference of the initial value of the iterative learning algorithm. Later, the ILC algorithm with initial value correction is proposed, and the ILC algorithm with fixed initial value deviation is better than the random one [11]. Based on the ILC algorithm of fixed initial value deviation analyzed by the above scholars, Sun Mingxuan proposed the ILC algorithm with initial value correction in 2014, but the algorithm needs to ensure that the initial deviation is bounded and cannot deal with the control problem with stochastic initial value deviation system [12]. For the system control problem with random initial value deviation, Lv Qing proposed an ILC algorithm with a modified interval, which decreases with the number of iterations [13]. The algorithm can effectively suppress the influence of the initial value deviation on the system output. In the PID algorithm, Park discusses the iterative learning control problem of linear system and a class of nonlinear systems with initial value deviation, and analyzes the influence of learning gain selection on the initial value deviation [14]. The iterative learning control problem with a fixed offset of the initial state and the expected initial state is discussed. A PD-type iterative learning control algorithm with feedback auxiliary is proposed to realize the asymptotic tracking of the expected trajectory of the system output. Li Yan et al. [15,16] extended the traditional iterative learning control time domain and frequency domain analysis methods to a class of fractional order nonlinear systems; they proposed a new type of fractional iterative learning control framework and simplified the convergence conditions, and solved the equivalence problem of two kinds of fractional order iterative learning control convergence conditions with the constant gain.

During continuous and repetitive experiments, if the initial conditions are reset every time, the practical results of the system test results are greatly reduced. In response to

this problem, in 2000, Korean scholar K Wang added a proportional term and an error integral term to the traditional D-type iterative learning control algorithm in order to reduce the influence of initial error on the control result [17,18]. In 1999, Chen used an initial state learning scheme combined with the traditional D-type iterative learning control algorithm to learn the law [19,20]. It may not be necessary to reset the initial conditions every time, and obtain the convergence boundary of the tracking error, depending on the uncertainty of the system and the external. The interference determines the convergence boundary without relying on the initial error. This paper proposes an iterative learning control method for accelerating the suppression of the random initial state error without reset condition, which is used to accelerate the suppression of initial random error. In 2013, Jiang Yue of Northeastern University proposed to increase the initial learning and design the D-type iterative learning control law with initial learning based on the original D-type learning law [21,22,23]. This method can eliminate the influence on the trajectory tracking effect of the initial migration. The downside is that the convergence speed of the algorithm is much lower than that prior to the initial state being learned [24,25].

Industrial robots make important content in the development process of our country, especially in the manufacturing industry [26]. Good stability, flexibility, high precision and high efficiency directly affect the efficiency and quality of the industry's products, and even affect the development process of the national manufacturing industry [27,28,29]. The stability of the manipulator system is an important research focus. The initial value conditions affect the convergence and convergence speed of the system. This paper is based on the PD type iterative learning control algorithm for the robotic arm system with external interference. The iterative learning control method for accelerating the suppression of random initial state errors is discussed. The influence of the method on the convergence and convergence speed of the manipulator system is analyzed.

2. PROBLEM STATEMENT AND CONTROLLER DESIGN

Consider the n-degree-of-freedom manipulator system dynamics model, the expression is as follows:

$$M(q_k)\ddot{q}_k + C(q_k, \dot{q}_k)\dot{q}_k + G(q_k) + d(t) = \tau(t) \quad (1)$$

where t denotes time, Non-negative integer k represents the number of iterations, \ddot{q}_k , \dot{q}_k , and $q_k \in R^n$ are acceleration, speed and position, respectively. $M(q_k) \in R^{n \times n}$ is inertia matrix, $C(q_k, \dot{q}_k) \in R^{n \times n}$ is the centrifugal force and the Coriolis force, $G(q_k) \in R^n$ is the gravity matrix, $d(t)$ is the external disturbance, and $\tau_k(t) \in R^n$ is the input torque.

The mechanical system dynamics model (1) has the following properties

Property 1 The inertia matrix $M(q_k)$ is positively bounded and satisfies the following conditions:

$$0 < \beta_1 < \|M(q_k)\| < \beta_2 \quad (2)$$

where $0 < \beta_1 < \beta_2$.

Property 2 The inertia matrix $M(q_k)$ satisfies the global Lipschitz continuous condition:

$$\|M(q_{k+1}) - M(q_k)\| \leq l_m \|q_{k+1} - q_k\| \quad (3)$$

where l_m is positive.

Property 3 $G(q_k)$ satisfies the global Lipschitz continuous condition:

$$\|G(q_{k+1}) - G(q_k)\| \leq g_m \|q_{k+1} - q_k\| \quad (4)$$

where g_m is positive.

Property 4 $C(q_k, \dot{q}_k)$ is bounded.

$$C(q_k, \dot{q}_k) \leq C_m \|\dot{q}_k\| \quad (5)$$

Lemma 1 Considering a continuous function $f(x, y)$, $x \in X$, $X = \{x \in R^p \mid |x| \leq \rho_i, 1 \leq i \leq p\}$, $\rho_i \geq 0$, so there exists $L(y)$, make that

$$\begin{aligned} \|f(\sigma(x_1), y) - f(\sigma(x_2), y)\| &\leq L(y) \|x_1 - x_2\|, \\ \forall x_1, x_2 \in R^p, \forall y \in R^q \end{aligned} \quad (6)$$

where $\|\sigma(x_1) - \sigma(x_2)\| \leq \|x_1 - x_2\|$.

Lemma 2 There exists $z(t) = [z_1(t), z_2(t), \dots, z_n(t)] \in R^n$, $t \in 0, T$, make that

$$\left(\int_0^t \|z(s)\| ds \right) e^{-\lambda t} \leq \frac{1}{\lambda} \|z(t)\|_\lambda \quad (7)$$

In this paper, we make the following basic assumptions:

Assumption 1 Gravity matrix $\|G(q_k)\|$ is bounded: $\|G(q_k)\| \leq l_g$, l_g is obtained from the actual limits of the system.

Assumption 2 $\dot{q}_k(t)$ is bounded, $\|\dot{q}_k(t)\| \leq V_m$, V_m is obtained from the actual limits of the system.

Assumption 3 External interferenced (t) is bounded, $\|d(t)\| \leq l_d$, l_d is obtained from the actual limits of the system.

For systems with random initial value deviations, we define a random error correction interval that varies with the number of iterations, and assume that the interval decreases as the number of iterations increases, thus reducing the time required to correct the random error. The controller is then designed to solve the problem whereby the robotic arm system needs to reset the initial conditions each time.

For a robotic arm system with random initial value deviation, we designed the following iterative control learning law:

$$\tau_{k+1}(t) = \tau_k(t) + M(q) [K_d \dot{e}_k + K_p e_k(t)] + \phi_k(t) X_k(0) \quad (8)$$

$$\phi_k = \begin{cases} e^{At} \left(\frac{2a^k}{h} \left(1 - \frac{a^k}{h} \right) \right), & t \in \left[0, \frac{h}{a^k} \right] \\ 0, & t \in \left[\frac{h}{a^k}, T \right] \end{cases} \quad (9)$$

$$X_k(0) = D e_k(0) + x_k(0) - x_{k+1}(0) \quad (10)$$

$$\dot{e}_{k+1}(0) = \dot{e}_k(0) + K_k e_k(0) \quad (11)$$

If $\|I_n - K_d\| < 1$, then the closed loop system is stable, and the following expression holds

$$\lim_{k \rightarrow \infty} (q_d(t) - q_k(t)) = \lim_{k \rightarrow \infty} (\dot{q}_d(t) - \dot{q}_k(t)) = 0 \quad (12)$$

where $e_k = q_d - q_k$, K_d, K_p , are symmetric positive definite matrix. $D = \|I_n - K_d\|^T$, $I_k \in R^{n \times n}$ is unit matrix.

3. CONVERGENCE ANALYSIS

For convenience, the time t will be omitted and defined as follows

$$M(q_k) \equiv M_k, G(q_k) \equiv G_k \quad (13)$$

According to (1), we have

$$\ddot{q}_k = M_k^{-1} \tau_k - M_k^{-1} [c(q, \dot{q}_k) \dot{q}_k + G(q_k) + d(t)] \quad (14)$$

$$x_k(t) = \begin{cases} x_{1k} = q_k(t) \\ x_{2k}(t) = \dot{q}_k(t) \end{cases} \quad (15)$$

where $x_k(t) = [x_{1k}^T(t) \ x_{2k}^T(t)]^T$. (14) and (15) yield

$$\dot{x}_k(t) = \begin{cases} \dot{x}_{1k}(t) = x_{2k}(t) \\ \dot{x}_{2k}(t) = M_k^{-1} \tau_k(t) - M_k^{-1} C(q_k, \dot{q}_k) \dot{q}_k + G_k + d(t) \end{cases} \quad (16)$$

so

$$x_k(t) = A x_k(t) + B \tau_k + \psi_k(t) \quad (17)$$

where $A = \begin{pmatrix} O_n & I_n \\ O_n & O_n \end{pmatrix}$, $A = \begin{pmatrix} O_n \\ M_k^{-1} \end{pmatrix}$, $\psi_k(t) = \begin{pmatrix} O_n \\ f_k(t) \end{pmatrix}$, $f_k(t) = -M_k^{-1} [C(q, \dot{q}_k) \dot{q}_k + G_k + d]$,

For arbitrary $\tau_k(t) =$, $t \in 0, T$, The general solution of (17) is:

$$x_k(t) = e^{At} x_k(0) + \int_0^t e^{A(t-s)} B \tau_k(s) ds + \int_0^t e^{A(t-s)} \psi_k(s) ds \quad (18)$$

where $e^{A(t-s)}$ is the state transition matrix of the system.

The introduction of (18) into (19), yields

$$\begin{aligned} x_{k+1} - x_k &= \int_0^t e^{A(t-s)} (\psi_{k+1}(s)) - \psi_k(s) ds \\ &+ \int_0^t e^{A(t-s)} \phi_k(s) X_k(0) ds \\ &+ \int_0^t e^{A(t-s)} B M_k K_d \dot{e}_k(s) ds \\ &+ \int_0^t e^{A(t-s)} B M_k K_p e_k(s) ds \\ &+ e^{At} (x_{k+1}(0) - x_k(0)) \\ &= \int_0^t e^{A(t-s)} (\psi_{k+1}(s) - \psi_k(s)) ds \\ &+ \int_0^t e^{A(t-s)} \phi_k(s) X_k(0) ds \\ &+ e^{At} [D e_k(0) - X_k(0)] \end{aligned}$$

$$\begin{aligned}
& + \int_0^t e^{A(t-s)} B M_k K_d \dot{e}_k(s) ds \\
& + \int_0^t e^{A(t-s)} B M_k K_p e_k(s) ds \quad (19)
\end{aligned}$$

where $B M_k K_d = D$, $B M_k K_p = P$, $D = [O_n \ K_d^T]^T$, $P = [O_n \ K_p^T]^T$
so

$$\begin{aligned}
x_{k+1}(t) - x_k(t) &= -e^{At} X_k(0) + \int_0^t A e^{A(t-s)} \varphi_k(s) X_k(0) ds \\
& + e^{At} D e_k(0) + \int_0^t e^{A(t-s)} P e_k(s) ds \\
& + \int_0^t e^{A(t-s)} \psi_{k+1}(s) - \psi_k(s) ds + \int_0^t e^{A(t-s)} D \dot{e}_k(s) ds \quad (20)
\end{aligned}$$

According to the step-by-step integral formula, by (9) and (20), we get

$$\begin{aligned}
x_{k+1}(t) - x_k(t) &= -D e_k(t) - \int_0^t A e^{A(t-s)} D e_k(s) ds \\
& - \int_0^t e^{A(t-s)} [\psi_{k+1}(s) - \psi_k(s)] ds \\
& - e^{At} X_k(0) \left[\int_0^t e^{-As} \varphi_k(s) ds - 1 \right] - \int_0^t e^{A(t-s)} P e_k(s) ds \quad (21)
\end{aligned}$$

The introduction of $x_d(t) = [q_d^T \ \dot{q}_d^T]^T$ into (21), yields

$$\begin{aligned}
E_{k+1}(t) - E_k(t) &= -D e_k(t) - \int_0^t A e^{A(t-s)} D e_k(s) ds \\
& - \int_0^t e^{A(t-s)} [\psi_{k+1}(s) - \psi_k(s)] ds \\
& - e^{At} X_k(0) \left[\int_0^t e^{-As} \varphi_k(s) ds - 1 \right] \\
& - \int_0^t e^{A(t-s)} P e_k(s) ds \quad (22)
\end{aligned}$$

where $E_k(t) = [e_k^T \ \dot{e}_k^T]^T$.

When $t \in [\frac{h}{a^k}, T]$, by (10), we get

$$e^{At} X_k(0) \left[\int_0^t e^{-As} \varphi_k(s) ds - 1 \right] = 0 \quad (23)$$

The introduction of (23) into (22), and taking the norm, yields

$$\begin{aligned}
\|E_{k+1}(t)\| &\leq \|E_k(t) - D e_k(t)\| + \int_0^t \|e^{A(t-s)}\| \|e_k(s)\| ds \\
& + \int_0^t \|e^{A(t-s)}\| \|P\| \|e_k(s)\| ds \\
& + \int_0^t \|e^{A(t-s)}\| \|\psi_{k+1}(s) - \psi_k(s)\| ds \quad (24)
\end{aligned}$$

as $E_k(t) = [e_k^T \ \dot{e}_k^T]^T$, obviously, $\|E_{k+1}(t) - D e_k(s)\| \leq \|I_2 - D\| \|E_k(t)\|$, $e_k(s) \leq \|E_k(t)\|$, where $I_2 = [I_n \ I_n]^2$

therefore,

$$\begin{aligned}
\|E_{k+1}(t)\| &\leq \|I_2 - D\| \|E_k(t)\| \\
& + \int_0^t \|e^{A(t-s)}\| \|AD\| \|E_k(s)\| ds \\
& + \int_0^t \|e^{A(t-s)}\| \|P\| \|E_k(s)\| ds \\
& + \int_0^t \|e^{A(t-s)}\| \|\psi_{k+1}(s) - \psi_k(s)\| ds \quad (25)
\end{aligned}$$

For convenience, we let

$$D(q_k, \dot{q}_k) = C(q_k, \dot{q}_k) \dot{q}_k \quad (26)$$

where $\left\| \frac{\partial}{\partial \dot{q}} D(q_k, \dot{q}_k) \right\| \leq \xi$

By (17) and (26), we have

$$\begin{aligned}
f_{k+1}(t) - f_k(t) &\leq - \left(M_{k+1}^{-1} - M_k^{-1} \right) G_k \\
& - M_{k+1}^{-1} (G_{k+1} - G_k) - \left(M_{k+1}^{-1} - M_k^{-1} \right) D(q_k, \dot{q}_k) \\
& - M_{k+1}^{-1} (D(q_k, \dot{q}_{k+1}) - D(q_k, \dot{q}_k)) - (M_{k+1}^{-1} - M_k^{-1}) d \quad (27)
\end{aligned}$$

According to (26) and (27), properties 1 to 4, lemma 1 and assumptions 1–2, yields

$$\begin{aligned}
f_{k+1}(t) - f_k(t) &\leq (l_m \beta_1^{-2} (C_m V_m^2 + l_g + l_d) + \beta_1^{-1} g_m) \\
& \times \|e_{k+1}(t) - e_k(t)\| + \beta_1^{-1} \xi \|\dot{e}_{k+1}(t) - \dot{e}_k(t)\| \quad (28)
\end{aligned}$$

so

$$f_{k+1}(t) - f_k(t) \leq \omega \|E_{k+1}(t) - E_k(t)\| \quad (29)$$

where $\omega = (l_m \beta_1^{-2} (C_m V_m^2 + l_g + l_d) + \beta_1^{-1} g_m) + \beta_1^{-1} \xi$

With (25) and (29), we obtain

$$\begin{aligned}
\|E_{k+1}(t)\| &\leq \|I_2 - D\| \|E_k(t)\| + \alpha \eta \int_0^t \|E_k(s)\| ds \\
& + \alpha \omega \int_0^t \|E_{k+1}(s) - E_k(s)\| ds \quad (30)
\end{aligned}$$

where $\sup_{t,s \in [0,T]} \|e^{A(t-s)}\|$ and $\eta = \|AD + P\|$

Multiply both sides of the (30) by $e^{-\lambda t}$, by the lemma 2, we obtain

$$\|E_{k+1}(t)\| \lambda \leq \frac{\|I_2 - D\| + \frac{\alpha \eta}{\lambda}}{1 - \frac{\alpha \omega}{\lambda}} \|E_k(t)\| \lambda \quad (31)$$

As $\|I_2 - D\| \leq 1$, $\|I_d - K_d\| \leq 1$, there is adequate λ ,

$$\frac{\|I_2 - D\| + \frac{\alpha \eta}{\lambda}}{1 - \frac{\alpha \omega}{\lambda}} = \rho < 1 \quad (32)$$

With (31), we obtain

$$\|E_{k+1}(t)\| \lambda \leq \rho \|E_k(t)\| \lambda \quad (33)$$

(32) and (33) yield

$$\lim_{k \rightarrow \infty} \|E_k(t)\| \lambda = 0, \quad t \in \left[\frac{h}{a^k}, T \right] \quad (34)$$

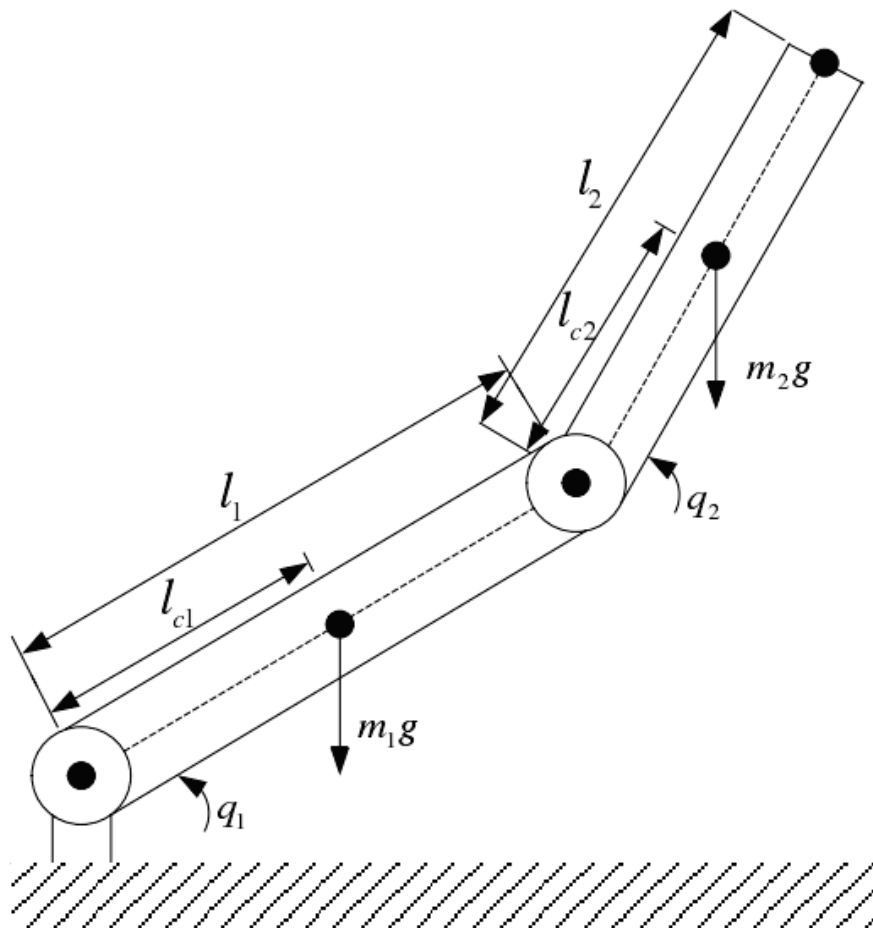


Figure 1 2-DOF manipulator system.

so, we obtain the following expression

$$\lim_{k \rightarrow \infty} (q_d(t) - q_k(t)) = \lim_{k \rightarrow \infty} (\dot{q}_d(t) - \dot{q}_k(t)) = 0 \quad (35)$$

From proof given above, we know that when the number of iterations k tend to infinity, the tracking error monotonically tends to zero at $t \in [\frac{h}{a^k}, T]$, if $t \in [0, \frac{h}{a^k}]$, then

$$\begin{aligned} \int_0^t e^{-As} \varphi_k(s) ds - 1 &= \int_0^t e^{-As} \cdot e^{As} \left[\frac{2a^k}{h} \left(1 - \frac{a^k}{h} t \right) \right] ds - 1 \\ &= \int_0^t \left[\frac{2a^k}{h} \left(1 - \frac{a^k}{h} t \right) \right] ds - 1 \\ &= - \left(\frac{a^k}{h} \right)^2 t^2 + 2 \left(\frac{a^k}{h} \right) t - 1 = - \left(\frac{a^k}{h} t - 1 \right)^2 \end{aligned} \quad (36)$$

Therefore, with the increasing of iteration times, the time will decrease when the robot arm cannot track the desired trajectory due to the initial error. When $t \in [0, \frac{h}{a^k}]$ and the iteration times k approaches infinity, $\frac{h}{a^k}$ will reduce, namely it can shorten the initial state correction time. In summary, for the dynamic model of the manipulator system for n degrees of freedom, we give the following introduction. Firstly, a correction interval (9) is defined on the time axis, which decreases as the number of iterations increases. Secondly, the condition (11) is given without resetting the initial value. Finally, an iterative learning control law is given according to equation (8). In terms of the λ norm, the actual output

of the final implementation system can completely track the expected output.

4. RESEARCH ON SIMULATION EXPERIMENT AND RESULT ANALYSIS

In order to verify the effectiveness of the iterative learning control algorithm proposed in this paper, MATLAB is used to simulate the trajectory tracking problem of a two-degree-of-freedom manipulator system. The two-degree-of-freedom manipulator system is shown in Figure 1.

We set the actual values of the parameters of the simulation experiment as follows

$$m_1 = 2Kg, m_2 = 2Kg, l_1 = 0.66m, l_2 = 0.6m, l_{c1} = 0.4, l_{c2} = 0.4, I_1 = 0.1, I_2 = 0.1$$

The desired trajectory is $q_{d1} = \frac{1}{2}\pi t - \sin(2\pi t)$, $q_{d2} = \frac{1}{2}\pi t - \sin(2\pi t)$.

This section simulates the robotic arm system under two different types of external disturbances.

(1) Sine function

The external disturbance is $d_1 = 0.2 \sin(30k\pi t)$, $d_2 = 0.2 \sin(30k\pi t)$.

We set the parameters for the simulation experiment as follows: $K_d = \text{diag}\{0.6, 0.6\}$, $K_p = \text{diag}\{5, 5\}$, $a =$

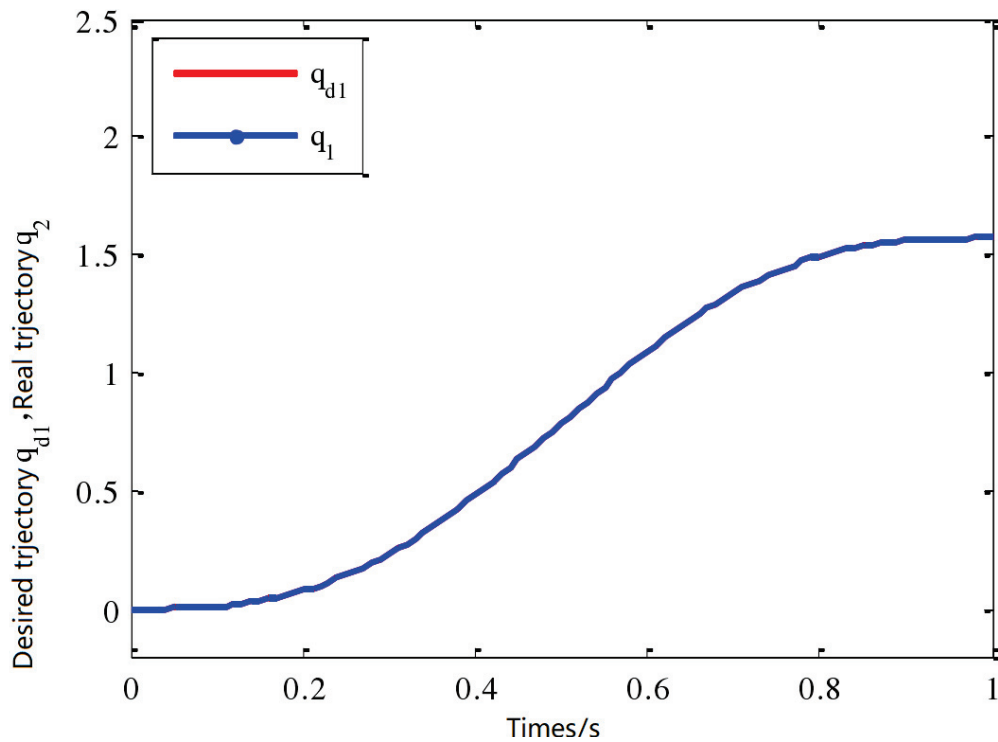


Figure 2 q_1 Tracking curve (the external disturbance of ILC is a sine function).

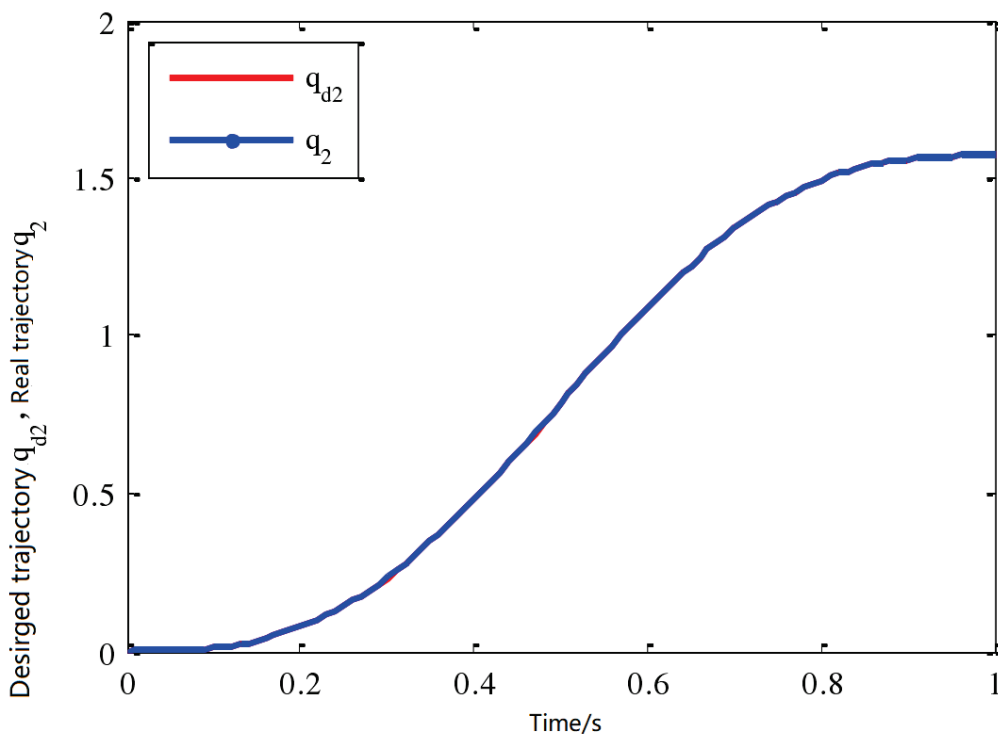


Figure 3 q_2 Tracking curve (the external disturbance of ILC is a sine function).

1.003, $h = 0.1$. The initial state values of system are randomly generated by the random function. When the external disturbance is a sinusoidal function, the simulation results of the mechanical arm system are shown in Figs. 2 to 5. Simulation experiments were performed on the tracking trajectory after 40 iterations of the manipulator, as shown in Figures 2 and 3.

Figure 2 and Figure 3 show the track traces of $1q$ and $2q$ at the 40th iteration, the solid line is the desired trajectory and the brokenline is the real-time trajectory. It can be seen from

the figures that the system output trajectory curve after the 40th iteration can completely track the desired trajectory. The learning law given in this paper has good control performance for the mechanical arm system with the external disturbance of the sinusoidal function.

Under the same conditions, the simulation experiments were carried out on the robotic arm system with acceleration suppression random initial state error and the mechanical arm system without acceleration suppression random initial state error.

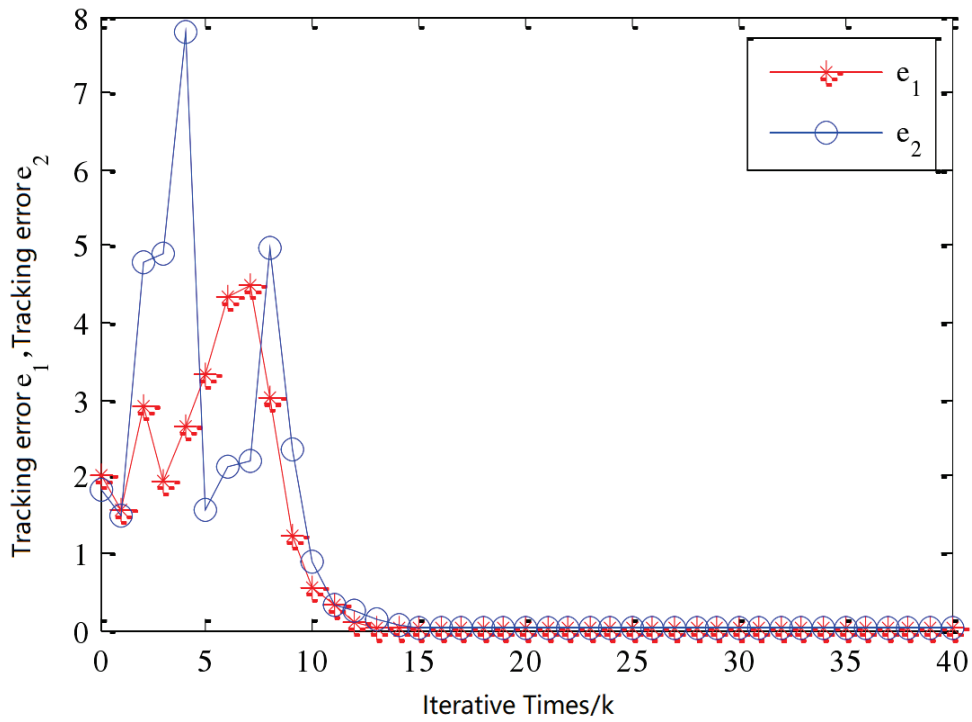


Figure 4 The tracking error cure with iterative numbers (the external disturbance of ILC is a sinusoidal function).

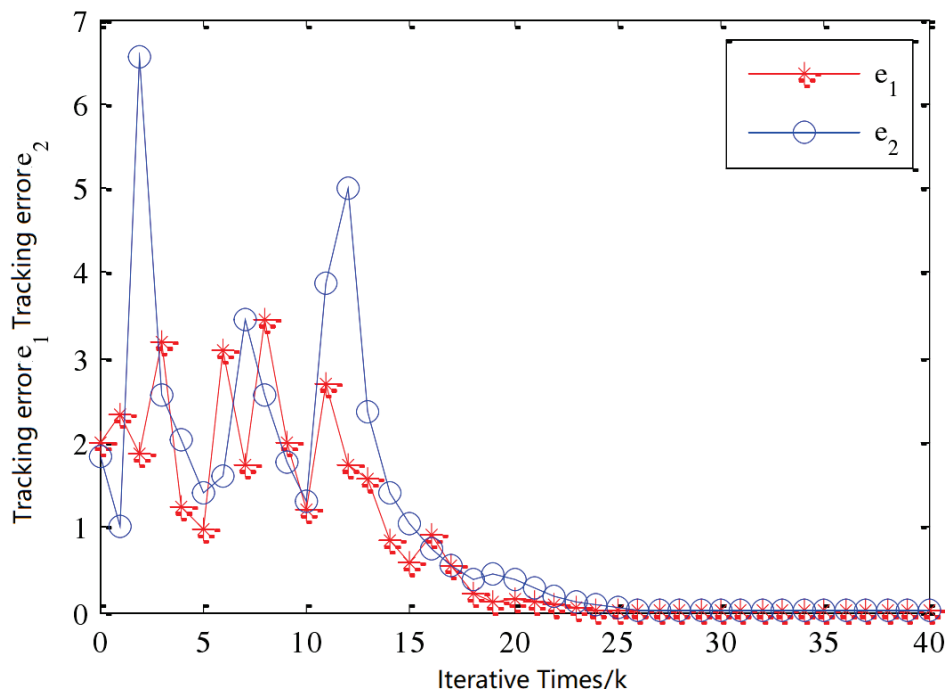


Figure 5 The tracking error cure with iterative numbers without acceleration-suppressing random errors (the external disturbance of ILC is a sinusoidal function).

In Figure 4, the curve with the asterisk is the first link tracking error (e_1) curve, and the circled curve is the second link tracking error (e_2) curve. It can be seen from the figure that the tracking error is small enough after 15 iterations, which indicates that the tracking error of the robot arm system is convergent. Figure 5 shows the ILC algorithm based on the non-acceleration-suppressed random initial state error. The tracking error of the manipulator system varies with the number of iterations. Comparing Figure 4

and Figure 5, it can be seen that under the acceleration suppression random initial error algorithm proposed in this paper, the tracking error can be reduced to zero around the 15th iteration. Under the ILC algorithm without acceleration suppression of random initial error, the tracking error is reduced to zero around the 25th iteration. Therefore, the iterative learning algorithm proposed in this paper has an obvious effect on accelerating the suppression of random errors.

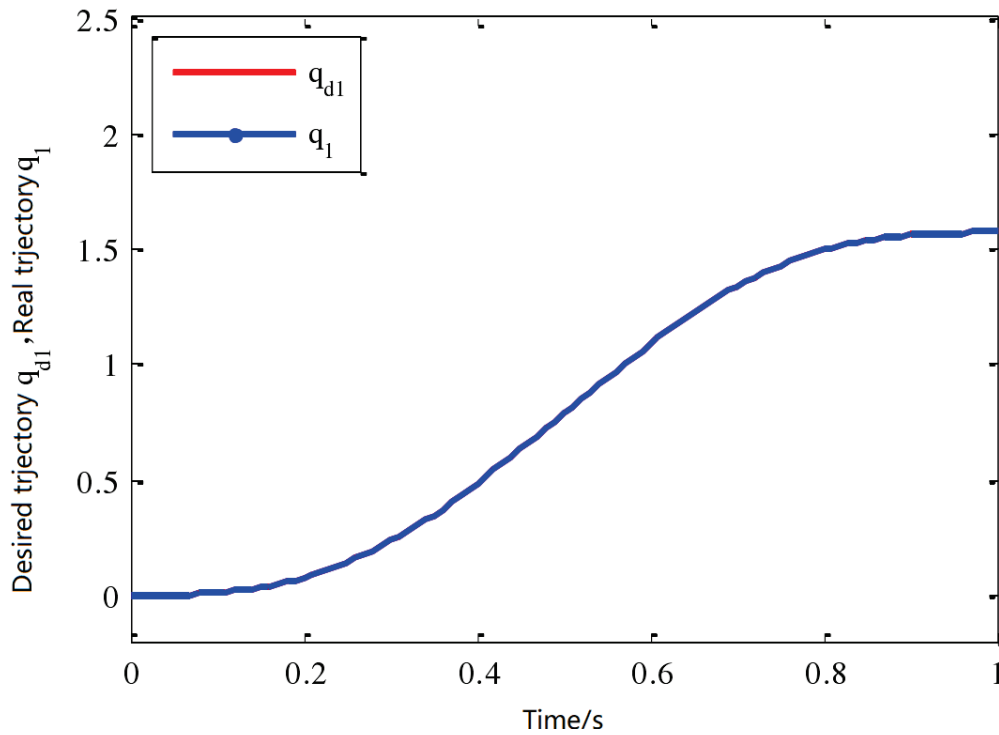


Figure 6 q_1 tracking curve (the external interference of ILC is a random function).

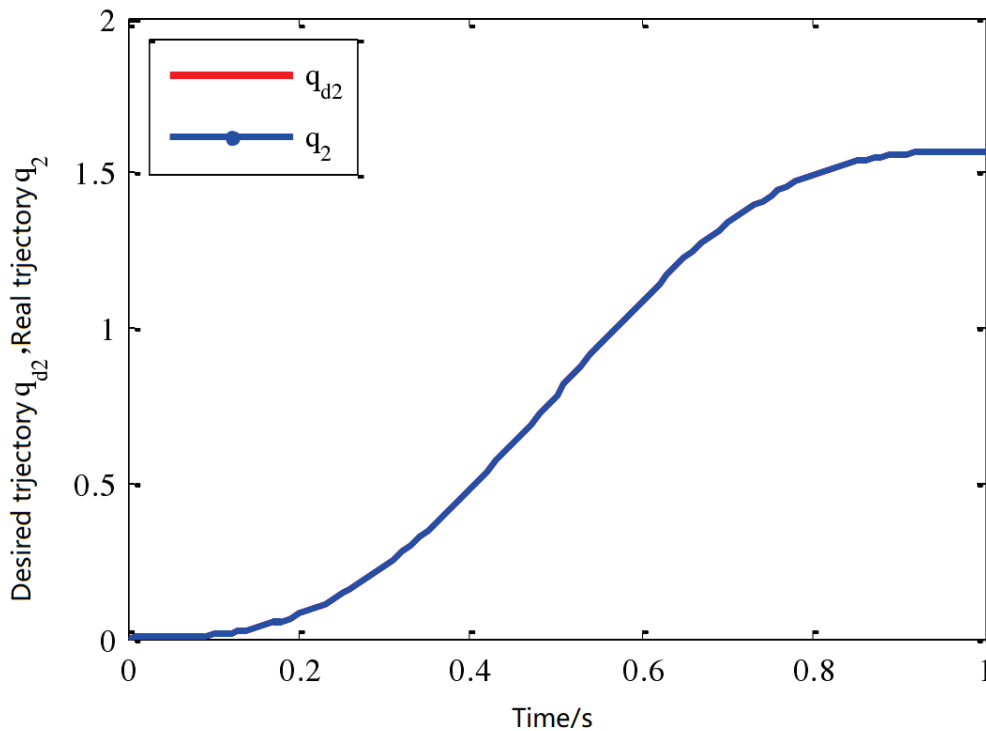


Figure 7 q_2 tracking curve (the external interference of ILC is a random function).

(2) Random function

The external disturbance is $d_1 = \text{random}(1)\sin(100k\pi t)$, $d_2 = \text{random}(1)\sin(100k\pi t)$.

We set the parameters of simulation experiment as follows: $K_d = \text{diag}\{0.4, 0.4\}$, $K_p = \text{diag}\{6, 6\}$, $a = 1.003$, $h = 0.1$. The initial state values of the system are randomly generated by the random function.

We control the trajectory tracking of each link of the manipulator. The simulation results of the manipulator system

when the external disturbance is a random function, are shown in Figures 6 to 9.

Figure 6 and Figure 7 show q_1 and q_2 track the trajectory curve at the 40th iteration; the solid line is the desired trajectory and the broken line is the real-time trajectory. As can be seen from the figure, the output trajectory can completely track the desired trajectory after the 40th iteration. The trajectory tracking control target has been implemented, illustrating that the ILC proposed in this paper is effective.

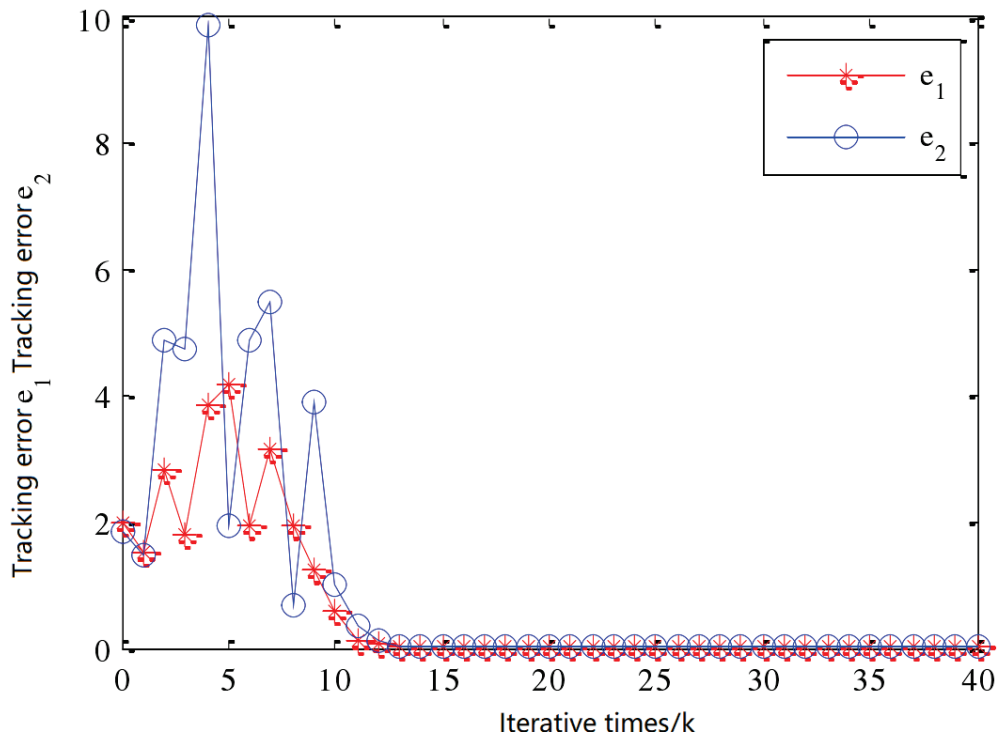


Figure 8 The tracking error cure with iterative numbers (the external disturbance of ILC is a random function).

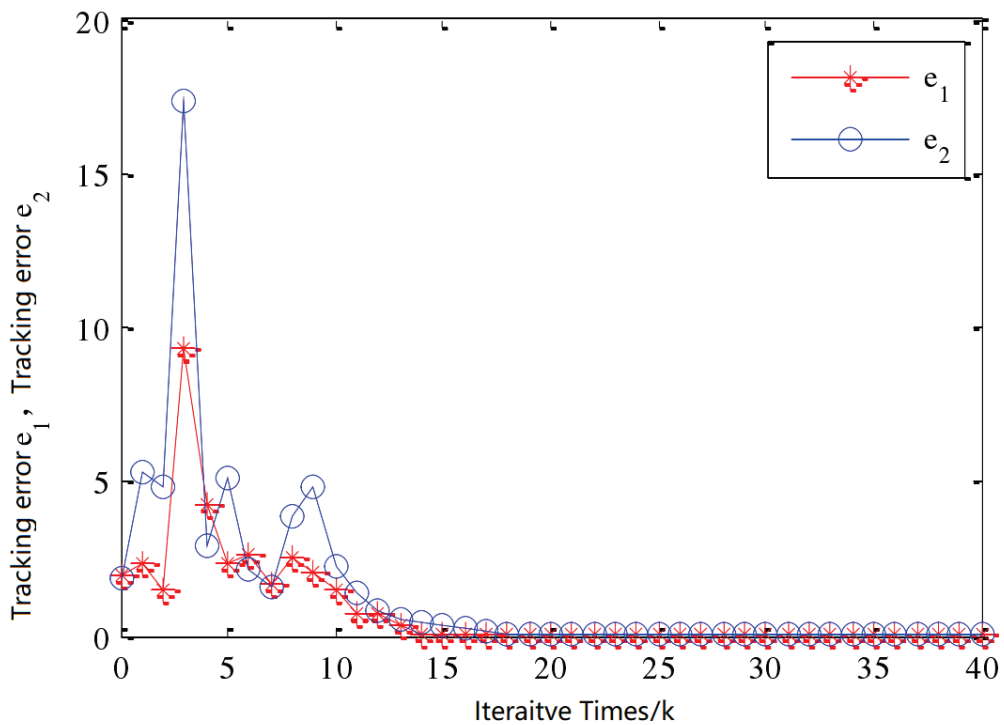


Figure 9 The tracking error cure with iterative numbers without acceleration-suppressing random errors(the external disturbance of ILC is a random function).

Under the same conditions, the simulation experiments were carried out on the robotic arm system with acceleration suppression random initial state error and the mechanical arm system without acceleration suppression random initial state error, as shown in Figure 8 and Figure 9.

Figure 8 shows the curve of the tracking error of the robot arm system with the number of iterations. The curve with the asterisk is the change of the first link tracking error (e_1), and the curve with the circle is the tracking error of

the second link (e_2). It can be seen from the figure that when the external disturbance is in the form of a random function, the tracking error has an initial oscillation, and after the 10th iteration, it starts to decrease with the number of iterations. After 13 iterations, the tracking error is small enough. It indicates the convergence of the tracking error of the robot arm system. Figure 9 shows the ILC algorithm based on the non-acceleration-suppressed random initial state. The tracking error of the two-degree-of-freedom manipulator

system varies with the number of iterations. As can be seen from Figure 8 and Figure 9, with the application of the acceleration suppression algorithm proposed in this paper, the tracking error can be reduced to zero around the 13th iteration. In the ILC algorithm without acceleration suppression, the tracking error is reduced to zero around the 19th iteration. Therefore, the proposed algorithm has an obvious effect on accelerating the suppression of random errors.

5. CONCLUSIONS

In this paper, the manipulator system with external disturbances is studied without resetting. Based on the PD-type iterative learning control algorithm, an iterative learning control method for accelerating the suppression of random initial errors is proposed. Firstly, a correction interval is defined on the time axis, which decreases as the number of iterations increases. Thereby, the algorithm eliminated the influence of the random initial state error on the robot arm. At the same time, the iterative learning control algorithm proposed in this paper obtains the initial value at each iteration without resetting. Secondly, under the λ -norm, the convergence of the algorithm is proved. Finally, when the external disturbance is a sinusoidal function and a random function, the proposed algorithm is compared with the iterative learning control under the condition of no acceleration suppression random error. The numerical simulation results show that the iterative learning control algorithm proposed in this paper can effectively accelerate and suppress random initial state errors. Therefore, the algorithm is effective.

ACKNOWLEDGMENTS

This work was supported by the Hechi University Foundation (XJ2016ZD004), Hechi university Youth teacher Foundation (XJ2017QN08), the Projection of Environment Master Foundation (2017HJA001, 2017HJB001), The important New Century Teaching Reform Project in Guangxi (2010JGZ033), Guangxi Youth Teacher Foundation (2018KY0495).

Authors' contributions

All authors contributed equally and significantly to the writing of this article. All authors read and approved the final manuscript. Mengji Chen is the corresponding author.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

REFERENCES

- Hua G Z, Ge Y X, Yang L. Value iteration-based H^∞ controller design for continuous-time nonlinear systems subject to input constraints[J]. *IEEE Transactions on Systems, Man, and Cybernetics Systems*, 2018. DOI: 10.1109/TSMC.2018.2853091.
- Wang L, Li X, Shen D. Sampled-data iterative learning control for continuous-time nonlinear systems with iteration-varying lengths[J]. *International Journal of Robust and Nonlinear Control*, 2018, 28(8): 3073–3091.
- Su H, Zhang H, Zhang K. Online reinforcement learning for a class of partially unknown continuous-time nonlinear systems via value iteration[J]. *Optimal Control Applications and Methods*, 2017, 39(2): 1011–1028.
- Zhu Q, Xu J X. Dual IM-based ILC scheme for linear discrete-time systems with iteration-varying reference[J]. *IET Control Theory & Applications*, 2018, 12(1): 129–139.
- Arimoto S, Nguyen P T A, Naniwa T. Learning of robot tasks on the basis of passivity and impedance concepts[J]. *Robotics and Autonomous Systems*, 2000, 32(2–3): 79–87.
- Wang Y, Liu T, Zhao Z. Advanced PI control with simple learning set-point design: Application on batch processes and robust stability analysis[J]. *Chemical Engineering Science*, 2012, 71(13): 153–165.
- Yan Q, Cai J, Wu L. Error-tracking iterative learning control for nonlinearly parametric time-delay systems with initial state errors[J]. *IEEE Access*, 2018, 6: 12167–12174.
- Tian Sheng Ping, Xu Huimin, Xie Shengli, et al. Iterative learning control algorithm for singular systems under initial state learning [J]. *Journal of China (NATURAL SCIENCE)*, 2017, 9(4): 411–416.
- Heinzinger G, Fenwick D, Paden B, Miyazaki F. Stability of learning control with disturbances and uncertain initial conditions[J]. *IEEE Transactions on Automatic Control*, 1992, 37(1),110–114.
- Zhang L, Chen W, Liu J, et al. A robust adaptive iterative learning control for trajectory tracking of permanent-magnet spherical actuator[J]. *IEEE Transactions on Industrial Electronics*, 2015, 63(1): 291–301.
- Lee D J, Park Y, Park Y S. Robust Hsliding mode descriptor observer for fault and output disturbance estimation of uncertain systems[J]. *IEEE Transactions on Automatic Control*, 2012, 57(11): 2928–2934.
- Sun Mingxuan, Bi Hongbo, Zhou Guoliang, Wang Huifeng. Feedback aided PD type iterative learning control: initial value problem and correction strategy [J]. *Automation Journal*, 2015, 4(01): 157–164.
- Lu Qing, Fang Yongchun, Ren Xiao. Iterative learning control for accelerating the suppression of random initial state error [J]. *Journal of Automation*, 2014, 40(07): 1295–1302.
- Park K H. An average operator-based PD-type iterative learning control for variable initial state error[J]. *IEEE Transactions on Automatic Control*, 2005, (6),865–869.
- Li Yan, Chen Yangquan, and Xiao Sheng. Convergence analysis of fractional order iterative learning control [J]. *Control Theory and Applications*, 2012, 29(08): 1031–1037.
- Bu X, Hou Z. Adaptive iterative learning control for linear systems with binary-valued observations[J]. *IEEE Transactions on Neural Networks & Learning Systems*, 2018, 29(1): 232–237.
- Wang K, Hyun P, Zeungnam B. A generalized iterative learning controller against initial state error[J]. *International Journal of Control*, 2000, 73(10): 871–881.
- Zhang R, Hou Z, Ji H. Adaptive iterative learning control for a class of non-linearly parameterised systems with input

- saturations[J]. *International Journal of Systems Science*, 2016, 47(5): 1084–1094.
19. Chen Y, Wen C, Gong Z. An iterative learning controller with initial state learning[J]. *IEEE Transactions on Automatic Control*, 1999, 44(2): 371–376.
 20. French M, Rogers E. Non-linear iterative learning by an adaptive Lyapunov technique [J]. *International Journal of Control*, 2000, 73(10): 840–850.
 21. Jiang Yue. Research on trajectory tracking control of robotic arm system based on iterative learning algorithm [D]. 2013.
 22. Gao Z, Liu X, Chen M Z Q. Unknown input observer-based robust fault estimation for systems corrupted by partially decoupled disturbances[J]. *IEEE Transactions on Industrial Electronics*, 2016, 63(4): 2537–2547.
 23. Grignion D, Chen X, Kar N. Estimation of load disturbance torque for DC motor drive systems under robustness and sensitivity consideration[J]. *IEEE Transactions on Industrial Electronics*, 2013, 61(2): 930–942.
 24. Ge X, Stein J L, Ersal T. Frequency-domain analysis of robust monotonic convergence of norm-optimal iterative learning control[J]. *IEEE Transactions on Control Systems Technology*, 2017, 26(2): 637–651.
 25. Zhao Y, Zhou F, Wang D. Path-tracking of mobile robot using feedback-aided P-type iterative learning control against initial state error[C]. Chongqing: 2017 6th Data Driven Control and Learning Systems, 2017.
 26. Wei J, Zhang Y, Sun M. Adaptive iterative learning control of a class of nonlinear time-delay systems with unknown backlash-like hysteresis input and control direction [J]. *ISA Transactions*, 2017, 70: 23–45.
 27. Jia X G, Yuan Z Y. Adaptive iterative learning control for robot manipulators[C]. *IEEE International Conference on Intelligent Computing and Intelligent Systems*, 2010: 1195–1203.
 28. Zhang R, Hou Z, Chi R. Adaptive iterative learning control for nonlinearly parameterised systems with unknown time-varying delays and input saturations [J]. *International Journal of Control*, 2015, 88(6): 1133–1141.
 29. Chi R H, Hou Z S, Jin S T. A data-driven adaptive ILC for a class of nonlinear discrete-time systems with random initial states and iteration-varying target trajectory[J]. *Journal of the Franklin Institute*, 2015, 352(6): 2407–2424.

