Eng Int Syst (2024) 6: 579–586

Engineering
Intelligent Systems

One-Dimensional Convection Diffusion Equation Based on Operator Splitting

Qinghua Yao[1], Benhua Qiu[2]* and Lina Chu[3]

[1] *Office of Academic Affairs, Xuchang Vocational College of Ceramic, Xuchang 461100, China*
[2] *Department of Basic Courses, Zhengzhou University of Science and Technology, Zhengzhou 450064, China*
[3] *Department of Basic Education, Chongqing Creation Vocational College, Chongqing 402160, China*

Due to the phenomenon of numerical dispersion and oscillation used for solving one-dimensional convection diffusion equations, the accuracy of numerical simulation results is not high. Therefore, a method is proposed based on operator splitting for a one-dimensional convection diffusion equation. Using the operator splitting algorithm, the undetermined coefficient method is applied to the convection and diffusion steps, and dimensionless coefficients are introduced to minimize the numerical oscillation and numerical diffusion of the scheme. A new numerical solution scheme of one-dimensional convection diffusion equation is constructed by using the results of convection step calculation as the known value to solve the diffusion equation. The experimental results show that the proposed method can effectively control the numerical oscillation and numerical diffusion, and has good convergence and stability, and high accuracy. It can effectively solve the one-dimensional convection diffusion equation, and has certain reference value.

Keywords: one-dimensional convection diffusion equation; solution; operator splitting; undetermined coefficient method

1. INTRODUCTION

The one-dimensional convection diffusion equation can be used to describe the convection and diffusion phenomena such as mass transfer and heat transfer in the atmospheric, oceanic, and river environments, and in the chemical industry [1]. Most solutions to fractional differential equations cannot be expressed in actual analytical form; That is to say, most analytical solutions are special functions that are difficult to calculate. However, with the wide application of fractional differential equations, how to improve the accuracy of numerical simulation results in the most convection diffusion problems, and how to solve the problem of how much computation and storage are required for fractional differential equations, has become a research focus of many scholars [2–3]. At present, there are several important numerical methods available for solving fractional differential equations. These include: finite difference method, finite element method, boundary element method and the characteristic line method [4–5]. At present, the convection diffusion type equation has been solved by using the differential operator theory and Euler Lagrange splitting scheme [6], and the undetermined coefficient method with high-order accuracy was constructed, which achieved good results in one-dimensional convection calculation [7]. However, these methods are used to solve the convection diffusion equation. In the optimal convection diffusion problem, numerical dispersion and oscillation are common, which affect the accuracy of numerical simulation results. In recent years, the operator splitting method has become an effective method for to solving the one-dimensional convection diffusion equation. Its main advantages are that the split equation is easier to solve, the scheme is flexible, and the stability is good. But it also has two shortcomings: one is that the splitting error is inevitable

*Email of corresponding author: qiubenhua2023@163.com

when the operator is not commutative; the other is to determine the intermediate boundary conditions of the splitting equation. The one-dimensional convection diffusion equation is divided into three equations (convection diffusion reaction), and the splitting error is analyzed theoretically [8]. We compared the solutions of one-dimensional convection diffusion equation under four different splitting schemes: standard Lie splitting, strang splitting (here the convection diffusion reaction term is divided into three operators), source splitting (here the convection and diffusion term is regarded as an operator, and the reaction term is regarded as an operator) and approximate matrix decomposition. There is room for improvement [9].

Hence, in this paper, a one-dimensional convection diffusion equation solution method based on operator splitting is proposed. The undetermined coefficient method is applied to the convection step and the diffusion step respectively, and a dimensionless coefficient is introduced into the scheme to minimize the numerical oscillation and numerical diffusion. A new numerical solution scheme for one-dimensional convection diffusion equation is constructed by using the convection step as the known value. The performance of the proposed method is tested by comparing it with that of other methods.

2. OPERATOR SPLITTING OF ONE-DIMENSIONAL CONVECTION DIFFUSION EQUATION

The one-dimensional convection diffusion equation describes the physical phenomenon of convection and anomalous sub-diffusion of particles or energy or other physical quantities in a physical system [10]. In this section, the operator splitting method for one-dimensional convection diffusion equation and its results are discussed.

Consider the following one-dimensional convection diffusion equation

$$\begin{aligned}&\frac{\vartheta c}{\vartheta t} + \Delta(u(x,t)c) = \nabla K \nabla c + R(c),\\ &x \epsilon \Omega\\ &t \epsilon T\\ &c|\vartheta\Omega = f(x,t)\end{aligned}$$

Where ω is a region $(d = 1, 2, 3)$ of $R(c)$, $u(x,t)$ is the velocity field, c is the solution concentration, x represents spatial variable, T is the temperature, K is the diffusion tensor. The reaction term, R, has different expressions in different cases. This leads to a complete and completely discrete solution process.

2.1 Operator splitting method

For A $\rightarrow$ B type Lie splitting. Generally, there are two splitting schemes:

1) Let $A = -u\nabla c + R(c)$, $B = \nabla(K\nabla c)$., so in each time step $[t_n, t_{n+1}]$, the following equation is solved:

$$\begin{aligned}&\frac{\vartheta c^1}{\vartheta t} = -u\nabla c^1 + R\left(c^1\right)\\ &c^1(x,t_n) = c(x,t_n)\\ &\frac{\vartheta c^2}{\vartheta t} = -u\nabla c^2 + R\left(c^2\right)\\ &c^2(x,t_n) = c^1(x,t_{n+1})\end{aligned}$$

It can be concluded that,

$$c(x,t_n) \approx c^2(x,t_{n+1})$$

2) Let $A = -u\nabla c + \nabla(K\nabla c)$, $B = R(c)$, so in each time step [tn, tn+1], the following equation is solved

$$\begin{aligned}&\frac{\vartheta c^3}{\vartheta t} = -u\nabla c + \nabla(K\nabla c^2)\\ &c^3(x,t_n) = c(x,t_n)\\ &\frac{\vartheta c^2}{\vartheta t} = R(c)\\ &c^2(x,t_n) = c^1(x,t_{n+1})\end{aligned}$$

It can be concluded that,

$$c(x,t_{n+1}) \approx c^2(x,t_{n+1})$$

2.2 Operator Classification Boundary

The one-dimensional convection diffusion equation essentially includes three simultaneous processes: convection, diffusion and reaction. The boundary conditions of the governing equation also simultaneously reflect the influence of these processes [11–12]. When two operators, A and B, are used to split the one-dimensional convection diffusion equation into union order, it is assumed that the two processes occur in sequence. Therefore, when the operator splitting method is used, it is very important to derive the boundary conditions suitable for splitting equations, i.e. the intermediate boundary conditions [13–14]. However, the boundary conditions of the operator splitting method have often led to "serious conflict". In this section, Dirichlet boundary conditions for the splitting Equations (3) and (4) are derived based on Leveque's concept of hyperbolic equation [15].

In order to simplify the problem, we consider the case that the velocity field u is time independent and $R(c) = \lambda C$. For u, it depends on time. For the case of $R(c)$ being nonlinear, we can use some iterative techniques. It turns into a linear case.

If Equation (3) is regarded as a Cauchy problem and integrated in a time step ΔT the following expression can be obtained

$$c^1(x,t_{n+1}) = \exp(\Delta t(A_1 + \lambda))c(x,t_n)$$

Here $A_1 = -u\nabla$, because we use the first order precision Lie splitting. Therefore, the first order approximation can also be made.

$$c^1(x,t_{n+1}) \approx c(x,(t_n) + t(A_1 + \lambda)c(x,t_n)$$

Let $f_1(t_{n+1})$ be an appropriate boundary condition for Equation (3); then the above equation can be expressed as

$$f^1(t_{n+1}) \approx c(x, t_n) + t(A_1 + \lambda)c(x, t_n)$$

and,
$x \epsilon \vartheta \Omega$
For Equation (4), the boundary conditions are derived on the assumption that f is sufficiently smooth.

$$f^2(t_{n+1}) = f(t_{n+\frac{1}{2}}, x)$$

At the end of each calculation cycle, the boundary conditions of the original equation are still used:

$$f^2(t_{n+1}) = f(t_{n+1}, x)$$

In Equation (7) $c(t_n, x) = f(t_n, x)$, where $c_x(t_n, x), c_y(t_n, x)$ is solved with the following equation $\frac{\vartheta c}{\vartheta t} = -u\nabla c + \lambda c$. Then, for equation $\frac{\vartheta c}{\vartheta t} = -u\nabla c + \lambda c$, with respect to the spatial boundary: □

$$\begin{cases} \frac{\vartheta c_x}{\vartheta t} = -u\nabla c_x + \lambda c_x \\ \frac{\vartheta c_y}{\vartheta t} = -u\nabla c_y + \lambda c_y \end{cases}$$

Where $\frac{\vartheta c}{\vartheta t}$ is the total derivative.

3. ONE-DIMENSIONAL CONVECTION DIFFUSION EQUATION BASED ON OPERATOR SPLITTING

On the basis of the above operator splitting results, the convection step and diffusion step are solved consecutively, and the results of each part are fused to reduce the error and improve the convergence.

3.1 Solution of Convection Step by Undetermined Coefficient Method

The Equation (2) is discretized by crank Nicolson scheme

$$\frac{u_{n+1} - u_n}{\Delta t} + \lambda(\frac{u_{n+1} - u_n}{2\Delta x}) = 0$$

where $\triangle t$ is the time step; $\triangle x$ is the space step. It can be obtained by changing Equation (4)

$$-\delta\frac{u_{n+1}}{4} + u_{n+1} + \delta(\frac{u_{n-1} - u_n}{4}) = \delta u_{n+1} + u_{n-1} - \delta\frac{u_n}{4}$$

where: $\delta = \frac{2\lambda\Delta t}{\Delta x}$ .I
In this scheme, algebraic averaging is used to discretize the convective terms, and the different roles of each node in the scheme are not considered. In order to improve the accuracy of the scheme, the undetermined coefficient method is used to assign different weight coefficients to each node [16–17]. At the same time, taking the minimum numerical oscillation and numerical diffusion of the scheme as the improvement objective, the appropriate values of these undetermined weight coefficients are determined. The coefficients in Equation (5) are replaced by undetermined coefficient $a_i (i = 1, 2, \ldots 6)$:

$$a_1 u_{n-1} + a_2 u + a_3 u_{n+1} = a_4 u_{n-1} + a_5 u_n + a_6 u_{n+1}$$

Using the Taylor expansion of Equation (16), it can be concluded that

$$\begin{aligned} &(a_1 + a_2 + a_3)\,\Delta t\frac{\vartheta u}{\vartheta t} + (a_2 + a_3 - a_1 - a_6)\,\Delta x\frac{\vartheta x}{\vartheta t} = \\ &\left[((a_4 + a_5 + a_6) - (a_1 + a_2 + a_3))\,u + \Delta x^2\right]\frac{\vartheta u}{\vartheta x} + \\ &(a_3 - a_1)\,\lambda\Delta t\Delta x\frac{\vartheta u}{\vartheta x} + (a_5 + a_6)\,\lambda\Delta t\Delta x^2\frac{\vartheta^2 u}{\vartheta^2 x} \\ &\quad + (a_2 + a_3)\,\lambda\Delta t\Delta x^3\frac{\vartheta^3 u}{\vartheta^3 x} \\ &\quad + (a_4 - a_2)\,\lambda\Delta t\Delta x^4\frac{\vartheta^4 u}{\vartheta^4 x} + (a_1 + a_6)\,\lambda\Delta t\Delta x^5\frac{\vartheta^5 u}{\vartheta^5 x} \\ &\quad + (a_4 - a_5)\,\lambda\Delta t\Delta x^6\frac{\vartheta^6 u}{\vartheta^6 x} \end{aligned}$$

Equation (17) is the equivalent differential equation of Equation (16) simulating convection Equation (2). In order to minimize the numerical oscillation and diffusion simultaneously, the following six independent algebraic equations are applied:

$$\begin{cases} a_1 + a_2 + a_3 = 1 \\ (a_3 + a_4) - (a_1 + a_6) = \delta \\ (a_4 + a_5 + a_6) - (a_1 + a_2 + a_3) = 0 \\ \delta^2 + 2\,(a_1 - a_3)\,\delta + (a_1 + a_3) - (a_4 + a_6) = 0 \\ \delta^3 - 3\,(a_1 - a_3)\,\delta^2 + 3\,(a_1 + a_3)\,\delta + (a_1 + a_3) \\ \quad - (a_4 + a_6) = 0 \\ \delta^4 - 4\,(a_1 - a_3)\,\delta^3 + 6\,(a_1 + a_3)\,\delta^2 + 4\,(a_1 - a_3)\,\delta \\ \quad + (a_1 + a_3) - (a_4 + a_6) = m \end{cases}$$

where m is a dimensionless parameter. This is based on the Fourier spectral analysis theory of numerical scheme stability analysis. When the first term on the right side of the equivalent differential equation is an odd order derivative term, the scheme is dominated by numerical dispersion; when it is an even order derivative term, the scheme has numerical diffusion property. A dimensionless parameter m is added to the right side of Equation (17) to enhance the stability of the scheme. The unique solution for the system of Equations (18) is

$$\begin{cases} a_1 = \frac{1}{12}(\delta - 1)(\delta - 2) + \frac{m}{4\delta(1+\delta)} \\ a_2 = \frac{1}{12}(\delta + 1)(\delta + 2) + \frac{m}{4\delta(\delta-1)} \\ a_3 = \frac{1}{12}(\delta + 1)(\delta + 2) + \frac{m}{4\delta(\delta-1)} \\ a_4 = \frac{1}{12}(\delta + 1)(\delta + 2) + \frac{m}{4\delta(\delta-1)} \\ a_5 = \frac{1}{12}(\delta + 1)(\delta + 2) + \frac{m}{4\delta(\delta-1)} \\ a_6 = \frac{1}{12}(\delta - 1)(\delta - 2) + \frac{m}{4\delta(1+\delta)} \end{cases}$$

3.2 Solution of Diffusion Step by Undetermined Coefficient Method

For the discretization of the same convection equation, the undetermined coefficient $b_i = (i = 1, 2, \ldots 6)$ is used to replace the coefficients in Equation (5)

$$b_1 u_{n-1} + b_2 u_{n+2} b_3 u_{n+1} = b_4 u_{n-1} + b_5 u_n + b_6 u_{n+1}$$

By Taylor expansion of Equation (16), it can be concluded that

$$
\begin{aligned}
&(b_1+b_2+b_3)\,\Delta t\frac{\vartheta u}{\vartheta t}+(b_2+b_3-b_1-b_6)\,\Delta x\frac{\vartheta x}{\vartheta t}= \\
&\left[((b_4+b_5+b_6)-(b_1+b_2+b_3))\,u+\Delta x^2\right]\frac{\vartheta u}{\vartheta x}+ \\
&(b_3-b_1)\,\lambda\Delta t\Delta x\frac{\vartheta u}{\vartheta x}+(b_5+b_6)\,\lambda\Delta t\Delta x^2\frac{\vartheta^2 u}{\vartheta^2 x} \\
&\quad+(b_2+b_3)\,\lambda\Delta t\Delta x^3\frac{\vartheta^3 u}{\vartheta^3 x} \\
&\quad+(b_4-b_2)\,\lambda\Delta t\Delta x^4\frac{\vartheta^4 u}{\vartheta^4 x}+(b_1+b_6)\,\lambda\Delta t\Delta x^5\frac{\vartheta^5 u}{\vartheta^5 x} \\
&\quad+(b_4-b_5)\,\lambda\Delta t\Delta x^6\frac{\vartheta^6 u}{\vartheta^6 x}
\end{aligned}
$$

Equation (21) is the equivalent differential equation of Equation (20) simulating diffusion Equation (3). In order to minimize both the numerical oscillation and the numerical diffusion of the scheme, the following six independent algebraic equations are applied.

$$
\begin{cases}
(b_1+b_2+b_3)\,\Delta x=1 \\
(b_3+b_4)-(b_1+b_6)=0 \\
(b_4+b_5+b_6)-(b_1+b_2+b_3)=0 \\
-((b_1+b_3)-(b_4+b_6))\,\frac{\Delta x^2}{4}=\tau \\
((b_3+b_4)-(b_1+b_6))\,\frac{\Delta x^3}{12}+(b_1-b_3)\,\Delta x\Delta t\tau=0 \\
2\,(b_1+b_2+b_3)\,\Delta t^2\tau^2+(b_1+b_3)\,\Delta x^2\Delta t\tau \\
+((b_1+b_3)-(b_4+b_6))\,\frac{\Delta x^4}{24}=0
\end{cases}
$$

The unique solution of the equations is

$$
\begin{cases}
b_1=\frac{1}{12\Delta t}-\frac{\tau}{\Delta x^2} \\
b_2=\frac{5}{6\Delta t}+\frac{2\tau}{\Delta x^2} \\
b_3=\frac{1}{12\Delta t}-\frac{\tau}{\Delta x^2} \\
b_4=\frac{1}{12\Delta t}+\frac{\tau}{\Delta x^2} \\
b_5=\frac{5}{6\Delta t}-\frac{2\tau}{\Delta x^2} \\
b_6=\frac{1}{12\Delta t}+\frac{\tau}{\Delta x^2}
\end{cases}
$$

3.3 Solution of Convection Diffusion Equation

In $[t_{n-1}, t_n]$ time steps, the results of convection step and expansion step are fused, and Equation (16) is solved by Equation (14); in $[t_{n-1}, t_n]$ time step, Equation (20) is solved by using the calculation results in $[t_{n-1}, t_n]$ time steps, which constitutes a complete step-by-step discrete solution process of convection diffusion equation (1). The following matrix forms can be obtained by combining Equations (19) and (23)

$$
\begin{cases}
AU_n=P_1U_{n-1}+F_1 \\
QU_{n+1}=P_2U_n+F_2
\end{cases}
$$

Among

$$
A=\begin{pmatrix} a_2 & a_3 & \\ a_1 & \ddots & \\ & a_1 & a_2 \end{pmatrix}
$$

$$
P_1=\begin{pmatrix} a_5 & a_6 & \\ a_4 & \ddots & \\ & a_5 & a_6 \end{pmatrix}
$$

$$
Q=\begin{pmatrix} b_2 & b_3 & \\ b_1 & \ddots & \\ & b_1 & b_2 \end{pmatrix}
$$

$$
P_1=\begin{pmatrix} b_5 & b_6 & \\ b_4 & \ddots & \\ & b_5 & b_6 \end{pmatrix}
$$

$$
U_n=[u_1,u_2,\dots u_n]^T
$$

$$
F_1=[(a_1-1)\,u_0,0,0,\dots,(a_3-1)\,u_n]^T
$$

$$
F_2=[(b_1-1)\,u_0,0,0,\dots,(b_3-1)\,u_n]^T
$$

3.4 One-Dimensional Convection Diffusion Equation Based on Operator Splitting

For (3), if

$$
\varphi(t,x)=\sqrt{1-\parallel u\parallel^2}
$$

The derivative operator of $\lambda=(\frac{u}{\varphi(t,x)},\frac{1}{\varphi(t,x)})$ is defined along the characteristic direction by using the special front method

$$
\frac{\vartheta}{\vartheta\lambda}=\frac{1}{\varphi(t,x)},\frac{\vartheta}{\vartheta t}+\nabla\frac{u}{\varphi(t,x)}
$$

When $u=(u_1,u_2,\dots u_d^2)$, $\parallel u\parallel^2=u_1^2+u_2^2+\dots u_d^2$. Then Equation (33) can be written as the following equivalent form

$$
\phi(t,x)(\frac{\vartheta c^1}{\vartheta\lambda})=R(c)^1
$$

If the abscissa of the intersection point of the inverse feature and the straight line $t=t_n$ is $\check{x}_{n-1}$ from point (t_{n-1},x), then at $t=t_{n-1}$, there is

$$
\varphi\left(\frac{\vartheta c^1}{\vartheta\lambda}\right)\approx\varphi\,(t_{n-1},x)\left(\frac{c^1\,(t_{n-1},x)-c^1\,(t_n,\check{x})}{(\|x-\check{x}_{n+1}\|)^2+(\Delta t)}\right.
$$

Here, $\check{x}$ can be determined by the characteristic line method. For any x, it is satisfied by the characteristic line of (t_{n+1},x).

$$
\frac{dx}{dt}=u
$$

$$
X=(t_{n-1},x,t_{n+1})=x
$$

According to the rectangular equation, integral Equation (35) can be obtained

$$
\check{x}_{n+1}=x-u(t_{n+1},x)\Delta t
$$

or

$$
u(-4y,4x)
$$

Using Equation (34), Equation (32) can be approximated by

$$\varphi(\frac{\vartheta c^1}{\vartheta \lambda}) \approx \frac{c^1(x, t_{n+1}) - c^1(\check{x}, t_n)}{\Delta t}$$

In the case of Equation (35), since there are $\check{x}_{n+1}$ and $u(t_n, \check{x}_{n+1})$ in (35), iterative solution is needed to determine $\check{x}_{n+1}$, such as Newton iterative method, so as to determine the value of $u(t_n, \check{x}_{n+1})$. Sometimes, the boundary may have been reached before the arrival of the time layer in the reverse direction.

$$\check{x}_{n+1} = x - u(t_n, x_{n+1})(t_{n+1} - t), t\epsilon \left[t_n, t_{n-1}\right]$$

or

$$\check{x}_{n+1} = x - u(t_n, x_{n+1})(t_{n+1} - t), t\epsilon \left[t_n, t_{n-1}\right]$$

In this case, Equation (39) can be approximated by the following equation

$$\varphi(\frac{\vartheta c^1}{\vartheta \lambda}) \approx \frac{c^1(x, t_{n+1}) - c^1(\check{x}, t_n)}{t_{n+1} - t}$$

Then the solution format is

$$\frac{c^1(x, t_{n+1}) - c^1(\check{x}, t_n)}{t_{n+1} - t} = R(c^1(x, t_{n+1}))$$

3.5 Error Analysis

In general, the splitting error is inevitable due to the non-commutativity of the operators. The splitting error based on Lie splitting can be expressed as

$$Error = -\frac{1}{2}\Delta t^2 AB\omega(c) = -\frac{1}{2}\Delta t^2 A'B't^3$$

Where A' and B' are Lie operators corresponding to operators A and B, and if operator A' is linear, then $A'_0 = A(c)$. For the convection step, the expression of the splitting error is

$$Error_1 = -\frac{1}{2}D(\Delta t - \Delta c)\Delta t^2 AB\omega(c) = -\frac{1}{2}\Delta t^2 A'B't^3$$

If the reaction term R is linear with respect to C and independent of X, then the error caused by splitting diffusion and reaction term can be eliminated; if D and u are independent of X, the error caused by splitting the convection and diffusion term can be avoided. If both of them hold, then there is no splitting error.

For the expansion, the expression of the splitting error is

$$Error_1 = -\frac{1}{2}D\Delta t^2 AB\omega(c) = -\frac{1}{2}\Delta t^2 A'B't^3$$

If $\omega u = 0$ and R do not depend on X, the error caused by splitting convection and reaction term can be eliminated; if the reaction term R is linear with respect to C and independent of X, the error caused by splitting diffusion and reaction term can be avoided. If both are true, the splitting error is zero.

4. EXAMPLE ANALYSIS

In order to verify the effectiveness of the proposed method, an example is given.

Example: $C_t + u\nabla c = \nabla(B\nabla c) + \lambda c$

The initial conditions are: $C(x, y) = \exp(-\frac{(x-x_0)^2+(y-y_0)^2}{2\alpha^2})$

Here, $(x_0, -y_0)$ and $\alpha > 0$ are the center and standard deviations, respectively. If the diffusion coefficient is greater than 0, the velocity field $u = (-4y, 4x)$, and λ is a constant, then its exact solution can be expressed as

$$C(x, y, t) = \frac{2\alpha^2}{2\alpha^2 + 4Bt}\exp(\lambda t - \frac{(x' - x_0)^2 + (y' - y_0)^2}{2\alpha^2 + 4Bt}$$

Where $(x', y', 0)$ is the intersection point of the characteristic line passing through the point (x, y, t) when $t = 0$,

$$x' = \cos(4t)x + \sin(4t)y$$
$$y' = -\sin(4t)x + \cos(4t)y$$

Parameter selection:

$$T = 1, \quad \Omega = [-1, 1]^2, \quad B = 0.01,$$
$$\lambda = 0.1, \quad (x_0, y_0) = (-0.5, -0.5), \quad \alpha^2 = 0.01$$

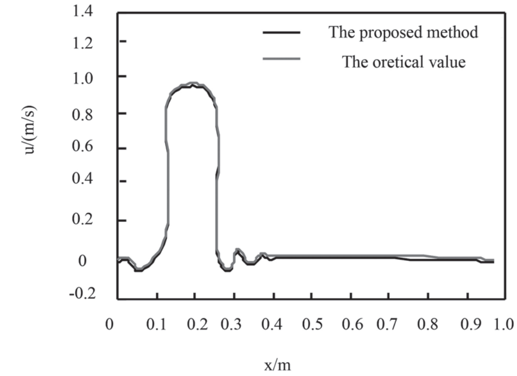
Figure 1 shows the results and corresponding theoretical solutions of the one-dimensional convection diffusion process when the dimensionless parameter m is different.

It can be seen from Figure 1 that when $m = 0$, the first term on the right side of the equivalent differential equation of the scheme is the fifth derivative term, and there is not enough numerical viscosity. Therefore, the calculation results have obvious numerical vibration. With the increase of M, the effect of numerical viscosity increases, while the effect of numerical vibration decreases. The ideal value of M is about 0.02-0.06, and the value of M is about 0.02-0.06 When it is less than 0.06, $M = 0.02$ should be selected for the actual calculation.

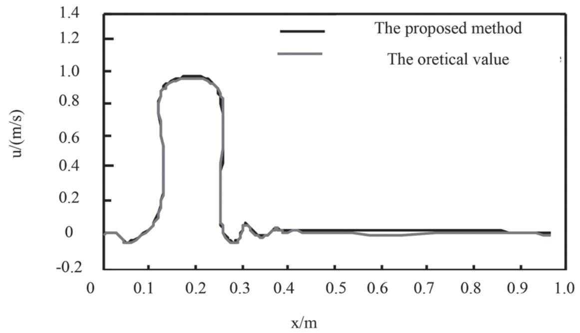
Comparing Figure 2 and Figure 1 (b), the upwind difference scheme has a strong numerical diffusion effect when simulating the convection and diffusion of sawtooth waves. It makes the initial sawtooth wave tip and slide at the top, and dissipates significantly from the bottom to both ends. The overall waveform tends to be plump. The results obtained are quite different from the theoretical solutions. The crank Nicolson scheme has obvious numerical oscillation, which is quite different from the theoretical solution. In contrast, the non-zero factor M is introduced in this method to achieve the actual physical process of material transport and diffusion and, at the same time, the control of numerical vibration and numerical diffusion can achieve ideal results.

In order to verify the convergence rate of the proposed method for the one-dimensional flow diffusion equation, the one-dimensional convection diffusion equation under different convection and fast three conditions are solved respectively. Results are reported below.

For the general convection diffusion equation, the diffusion coefficient $k = 0.1$. The convergence with different convection coefficients is shown in Figure 3 and Figure 4.



(a) m=0



(b) m=0.02

Figure 1 Comparison of calculation results and theoretical solutions.

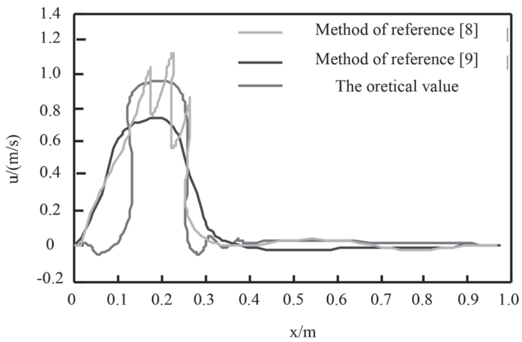


Figure 2 Comparison of calculation results.

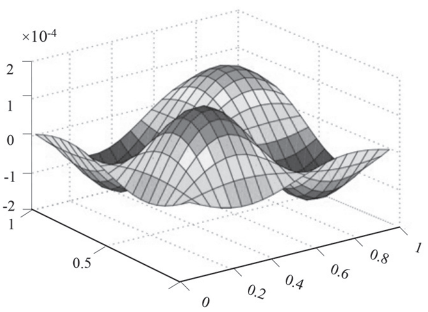


Figure 3 Error chart.

Figure 5 and Figure 6 show the calculation for the convergence of diffusion in the convection-dominated case.

It can be seen from the Figures 5 and 6 that the proposed method has good convergence under different diffusion and convection conditions. This is mainly because the convection step and the expansion calculation are not discretized separately in the operator separation stage, and the calculation results of the two are integrated into the calculation process of the other party in order to maximize the elimination of error, and improve the convergence of the calculation results.

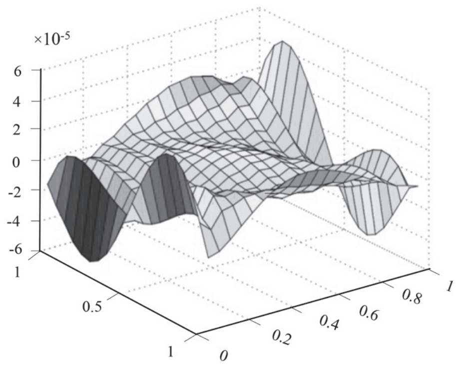


Figure 4 Error chart.

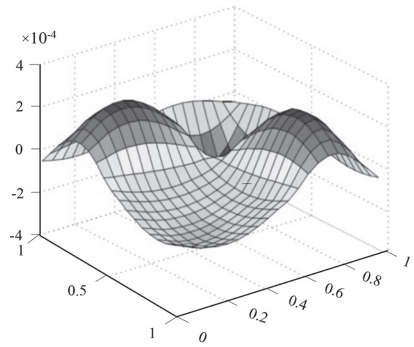


Figure 5 Error chart.

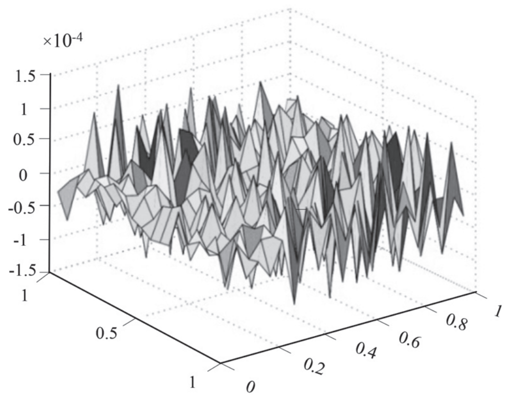


Figure 6 Error chart.

5. CONCLUSION

In regard to the numerical dispersion and oscillation phenomenon in the one-dimensional convection diffusion equation, a method based on operator splitting is proposed. By using an undetermined coefficient method for convection step and diffusion step respectively, dimensionless coefficients are introduced into the scheme according to the minimum numerical oscillation and numerical diffusion. The numerical solution scheme of the one-dimensional convection diffusion equation is constructed by using the convection step as the known value and applying this to the solution of the diffusion equation. The experimental results show that this method can effectively control numerical oscillation and diffusion, and has good convergence and stability, which is valuable for related research in the future.

At present, the research is limited to the solution of a one-dimensional convection diffusion equation. In the later research, we can broaden the research scope and study more multidimensional convection diffusion equations.

Data Availability Statement

The datasets used and/or analyzed during the current study are available from the corresponding author on reasonable request.

Conflicts of Interest

It is declared by the authors that this article is free of conflict of interest.

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